Merging DEL and ETL for epistemic planning

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Dagstuhl, January 15th, 2014
1. DEL and ETL: the marriage via axioms

2. A mixed-blood baby for contingent planning

3. Happy ending?
Background

Two modal logic approaches handling knowledge and actions:

- **Epistemic** Temporal Logic (**ETL**): knowledge in distributed systems based on **temporal logic**.
  [Fagin et al., 1995, Parikh and Ramanujam, 1985]

- **Dynamic** Epistemic Logic (**DEL**): knowledge in multi-agent interactions based on **epistemic logic**.
They are semantics-driven two-dimensional modal logics:

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<td>DEL</td>
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\[ \neg Kp \land EF Kp \]

\[ \neg Kp \land [!p]Kp \]

\[ \begin{array}{c}
\circ\bullet p \\
\downarrow p \\
\downarrow \neg p
\end{array} \]

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Dynamic semantics:

The **meaning** of an event is the **change** it brings to the knowledge states (dates back to [Stalnaker, 1978]).
Bridging the two

An earlier observation: Iterated updating epistemic structures generates special ETL-style “super models” [van Benthem et al., 2009].

Our approach: relate the two via axioms.


- New axiomatizations of PAL/DEL using ETL-style axioms
- ETL-style completeness proof method for DEL-style logics
- Characterization results of product update and DEL-generatable ETL models.
- New axiomatization of DEL with protocols (no reduction possible)

Dynamic Epistemic Language (LDEL)

\[ \phi ::= \top \mid p \mid \neg \phi \mid \phi \land \phi \mid \Box \phi \mid [e]\phi \]

where \( p \in \mathbf{P} \) and \( e \in \Sigma \).

Given an (epistemic) model \( M = (S, \rightarrow, V) \) and a fixed event model \( U \), the semantics is as follows ([Baltag et al., 1998]):

\[
M, s \models \Box \psi \iff \forall t : s \rightarrow t \text{ implies } M, t \models \psi \\
M, s \models [e]\phi \iff M, s \models \text{Pre}(e) \text{ implies } M \otimes U, (s, e) \models \phi
\]
Reduction-to-static-based axiomatization

System $\mathcal{DE}$ (without Uni. sub. )

- **TAUT**: all the instances of tautologies
  
- **DISTK**: $\Box(\phi \rightarrow \psi) \rightarrow (\Box \phi \rightarrow \Box \psi)$
  
- **UATOM**: $[e]p \leftrightarrow (Pre(e) \rightarrow p)$
  
- **UNEG**: $[e]\neg \phi \leftrightarrow (Pre(e) \rightarrow \neg [e]\phi)$
  
- **UCON**: $[e](\phi \land \chi) \leftrightarrow ([e]\phi \land [e]\chi)$
  
- **UK**: $[e]\Box \phi \leftrightarrow (Pre(e) \rightarrow \bigwedge_{f:e \rightarrow f} \Box [f]\phi)$

Proof of completeness via reduction to basic modal logic $K$:

$$\models \phi \iff \models t(\phi) \implies \vdash_K t(\phi) \implies \vdash_{\mathcal{DE}} t(\phi) \implies \vdash_{\mathcal{DE}} \phi.$$ 

It does not come free. Be careful! [Wang, 2011]
**New axiomatization (given image-finiteness of $U$)**

**System $\mathcal{DEN}$**

<table>
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<td><strong>MP</strong></td>
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<td>all the instances of tautologies</td>
<td>$\phi, \phi \rightarrow \psi$</td>
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<tr>
<td><strong>DISTK</strong></td>
<td><strong>NECK</strong></td>
</tr>
<tr>
<td>$\square(\phi \rightarrow \chi) \rightarrow (\square \phi \rightarrow \square \chi)$</td>
<td>$\square \phi$</td>
</tr>
<tr>
<td><strong>DISTU</strong></td>
<td><strong>NECU</strong></td>
</tr>
<tr>
<td>$[e](\phi \rightarrow \chi) \rightarrow ([e] \phi \rightarrow [e] \chi)$</td>
<td>$[e] \phi$</td>
</tr>
<tr>
<td><strong>INV</strong></td>
<td></td>
</tr>
<tr>
<td>$(p \rightarrow [e]p) \land (\neg p \rightarrow [e] \neg p)$</td>
<td></td>
</tr>
<tr>
<td><strong>PRE</strong></td>
<td></td>
</tr>
<tr>
<td>$\langle e \rangle \top \leftrightarrow Pre(e)$</td>
<td></td>
</tr>
<tr>
<td><strong>SNM</strong></td>
<td></td>
</tr>
<tr>
<td>$\Diamond \langle f \rangle \phi \rightarrow [e] \Diamond \phi$ (if $e \leftrightarrow f$ in $U$)</td>
<td></td>
</tr>
<tr>
<td><strong>SPR</strong></td>
<td></td>
</tr>
<tr>
<td>$\langle e \rangle \Diamond \phi \rightarrow \bigvee_{f : e \rightarrow f} \Diamond \langle f \rangle \phi$</td>
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As a special case for Public Announcement Logic:

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<td>$[\psi](\phi \rightarrow \chi) \rightarrow ([\psi] \phi \rightarrow [\psi] \chi)$</td>
</tr>
<tr>
<td>INV</td>
<td>$(\rho \rightarrow [\psi] \rho) \land (\neg \rho \rightarrow [\psi] \neg \rho)$</td>
</tr>
<tr>
<td>PRE</td>
<td>$\langle \psi \rangle T \leftrightarrow \psi$</td>
</tr>
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<td>$\langle \psi \rangle \Box \phi \rightarrow \Box [\psi] \phi$</td>
</tr>
<tr>
<td>SPR</td>
<td>$\Box [\psi] \phi \rightarrow [\psi] \Box \phi$</td>
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We can derive functionality ($\langle \psi \rangle \phi \rightarrow [\psi] \phi$) (but very complicated).

SNM is a (conditional) version of no learning $([\psi] \Box \phi \rightarrow \Box [\psi] \phi)$.
New proof method

Basic idea: treat $[e]$ as a **standard** modality interpreted on the standard two-dimensional ETL models with labelled events:

$$(S, \rightarrow, \{\rightarrow| e \in \Sigma\}, V)$$

$$\mathcal{M}, s \models [e] \phi \iff \forall t : s \xrightarrow{e} t \text{ implies } \mathcal{M}, t \models \phi$$

Proof strategy: find a class of ETL-style models $\mathcal{C}$ and show the following:

$$\equiv \phi \iff \mathcal{C} \models \phi \iff \vdash_{\mathcal{DE}N} \phi.$$ 

The first $\iff$ is proved by **flattening** the dynamics in the models in $\mathcal{C}$. 
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The class $C$: **normal** ETL models

**PRE:** $\langle e \rangle \top \leftrightarrow Pre(e)$,

**INV:** $(p \rightarrow [e]p) \land (\neg p \rightarrow [e]\neg p)$,

**SPR:** $\langle e \rangle \diamond \varphi \rightarrow \bigvee_{f:e \rightarrow f} \langle f \rangle \varphi$,

**SNM:** $\varnothing \langle f \rangle \varphi \rightarrow [e] \diamond \varphi$ (if $e \epsilon f$).

**Pre** $s$ has $e$-successors iff $N, s \models Pre(e)$.

**Inv** if $s \xrightarrow{e} t$ then for all $p \in P : t \in V(p) \iff s \in V(p)$.

**Nm** if $s \rightarrow s'$ and $s' \xrightarrow{f} t'$ then for all $e$ and $t$ such that $s \rightarrow t$ and $e \epsilon f$, we have $t \rightarrow t'$.

**Pr** if $s \xrightarrow{e} t$ and $t \rightarrow t'$ then there exists an $s'$ such that $s \rightarrow s'$ and $s' \xrightarrow{f} t'$ for some $f$ such that $e \epsilon f$ in $U$.

```
s \xrightarrow{e} s'  SNM  s \xrightarrow{e} s'  SPR  s
\downarrow e  \downarrow t  \downarrow e  \downarrow t  \downarrow e
\downarrow t  \rightarrow t'  \downarrow t  \rightarrow t'  \downarrow t  \rightarrow t'
```
Some applications

**Theorem**

DEL with protocols in axiomatized by replacing \( PRE \) with:

\[
\text{PPRE} : \langle e \rangle \top \rightarrow \text{Pre}(e) \quad \text{and} \quad \text{DET} : \langle e \rangle \langle h \rangle \top \rightarrow [e]\langle h \rangle \top
\]

The proof system is equivalent to the system of [Hoshi and Yap, 2009].

**Theorem**

\( SNM, SPR, INV \) and \( PRE \) characterize the update product operation.

Similar result: [van Benthem, 2011, Ch 3.8] on PAL.

**Theorem**

\( \text{Nm, Pr, Inv} \) and \( \text{Pre} \) characterize the product update generatable image-finite ETL models.
Half-way summary

- DEL-like logics can be viewed as special ETL logics.
- A completeness method for DEL-like logics via ETL.
- Same axioms, two views (global vs. constructive).
- Reduction-to-statics: a beautiful coincidence.
- Classic results for two-dimensional logics may be adapted.

Research agenda: meaningful ETL properties $\implies$ dynamic operators $\implies$ sweet-spot logics

There is a whole new world beyond the reduction-to-statics programme!
### A mixed-blood approach

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PRE, INV, and determinacy axiom are kicked out, thus reduction is impossible, but it does not matter at all.

The basic framework: *Not all those who wander are lost* [Wang and Li, 2012]: contingent planning.
Lost with a map at hand
Lost with a map at hand
Scenario in *Mission Impossible*

The secret agent sneaking in an enemy building is guided by his headquarters. Usually the communication with the HQ will be lost at some point, then the agent needs to find his own way.

$$\begin{align*}
S_1 \rightarrow & S_2 \rightarrow S_3 \rightarrow S_4: \text{Safe} \rightarrow S_5 \\
S_7 \leftarrow & S_6 \quad S_8: \text{Safe} \quad S_9: \text{Safe}
\end{align*}$$
Scenario in *Mission Impossible*

- HQ can guide the agent to move right, but the agent may not know that he is safe.
- HQ can guide the agent to move up, and the agent should know that he is safe.
- Agent may plan to move right then up to guarantee his safety.
Model: Kripke model with an uncertainty set

Given a set $P$ of basic propositions and a set $\Sigma$ of basic actions:

- A (non-deterministic) transition system:
  
  $\mathcal{N} = \langle S, \{R_a | a \in \Sigma\}, V \rangle$
  
  where $S \neq \emptyset$, $R_a \subseteq S \times S$ and $V : P \rightarrow 2^P$.

- An uncertainty map (UM):
  
  $\mathcal{M} = \langle S, \{R_a | a \in \Sigma\}, V, Q \rangle$
  
  where $Q \neq \emptyset$, $Q \subseteq S$ such that $\forall s, t \in S, e(s) = e(t)$.

- $\mathcal{M}, s$ is a pointed UM model, if $s \in Q$.

---

Example $(\mathcal{M}, s_3)$

\[
\begin{align*}
S_7 & \xleftarrow{l} S_6 & S_8: \text{Safe} & S_9: \text{Safe} \\
S_1 & \xrightarrow{r} S_2 \xrightarrow{r} S_3 \xrightarrow{r} S_4: \text{Safe} & \xrightarrow{r} S_5
\end{align*}
\]
Epistemic Action Language

- EAL language with action and knowledge as modalities:

  $$\phi ::= \top | p | \neg \phi | \phi \land \phi | \langle a \rangle \phi | K\phi$$

  where $p \in P$, $a \in \Sigma$.

- For abbreviations: $\bot := \neg \top$, $\phi \lor \psi := \neg (\neg \phi \land \neg \psi)$, $\phi \rightarrow \psi := \neg \phi \lor \psi$, $[a]\phi := \neg \langle a \rangle \neg \phi$, $\hat{K}\phi := \neg K\neg \phi$.

- The intuition of the formula:
  - $K\phi$ says that the agent knows that $\phi$
  - $\langle a \rangle \phi$ says that it is possible that after doing $a$, $\phi$ holds.
Given any UM model $\mathcal{M} = \langle S, \{R_a \mid a \in \Sigma \}, V, Q \rangle$, the satisfaction relation on pointed UM model $\mathcal{M}$, $s$ is defined as:

$\mathcal{M}, s \models K\phi \iff \forall u \in Q : \mathcal{M}, u \models \phi$

$\mathcal{M}, s \models \langle a \rangle\phi \iff \exists t \in S$ such that $s \xrightarrow{a} t$ and $\mathcal{M}|_t^a, t \models \phi$

- $\mathcal{M}|_t^a = \langle S, \{R_a \mid a \in \Sigma \}, V, Q|_t^a \rangle$
- $Q|_t^a = Q|_t^a \cap E(t)$ ‘carry’ the circle along $a \rightarrow$, and then check with the observations at the current location
- $Q|_t^a = \{r' \mid \exists r \in Q$ such that $r \xrightarrow{a} r'\}$
- $E(t) = \{t' \mid e(t') = e(t)\}$
Example

\[ \mathcal{M}, s_1 \models K \neg p \land \langle b \rangle \neg Kp \text{ and } \mathcal{M}_{s_3}^b, s_3 \models \neg Kp \]

Example (path dependency)

\[ \mathcal{M}, s_1 \models K \neg p \land \langle a \rangle \langle a \rangle Kp \text{ and } (\mathcal{M}_{s_2}^a)^a, s_3 \models Kp \]
The scenario in *Mission Impossible*

$$
\begin{align*}
S_7 & \leftarrow l \rightarrow S_6 \\
& \uparrow u \\
S_8 & \rightarrow S_9 \\
& \uparrow u \\
S_1 & \rightarrow r \rightarrow S_2 \rightarrow r \rightarrow S_3 \rightarrow r \rightarrow S_4 \rightarrow S_5 \\
& \uparrow u \\
\end{align*}
$$

- $M, s_3 \not\models \langle r \rangle (\text{Safe} \land \neg K\text{Safe})$
  (HQ guides you safe but you do not know it)
- $M, s_3 \not\models \langle u \rangle (\text{Safe} \land K\text{Safe}) \land \neg K[u]\text{Safe}$
  (HQ guides you safe and you know it)
- $M, s_3 \not\models K([r][u]K\text{Safe} \land \langle r \rangle \langle u \rangle K\text{Safe})$
  (You know the plan will make you safe)

We can use our framework to **verify** whether a plan can guarantee a goal via model checking.
A sound and complete axiomatization $S_{EAL}$

**Axioms:**

- **TAUT** all the axioms of propositional logic
- **DISTK** $K(p \rightarrow q) \rightarrow (KP \rightarrow Kq)$
- **DISTUa** $[a](p \rightarrow q) \rightarrow ([a]p \rightarrow [a]q)$
- **T** $KP \rightarrow p$
- **4** $KP \rightarrow KKP$
- **5** $\neg KP \rightarrow K\neg KP$
- **OBS(a)** $K\langle a \rangle \top \lor K\neg \langle a \rangle \top$
- **SPR** $\langle a \rangle \hat{K}p \rightarrow \hat{K}\langle a \rangle p$
- **SNM** $\land_{B \subseteq \Sigma}(\hat{K}\langle a \rangle(p \land \psi_B) \rightarrow [a](\psi_B \rightarrow \hat{K}p))$

**Rules:**

- **MP** **NECK** **NECU** **SUB**

This proof system can be used to do syntactic reasoning.
Some theoretical results

- Normal form: K can be pushed to the outermost position
- Finite model property and decidability
- A bisimulation notion satisfying Hennessy-Milner property

The features:
⟨simple language, intuitive semantics, compact models⟩.

Ongoing work: model checking complexity (with Quan Yu).
One child policy has been relaxed: extensions

\[ \phi ::= \top | p | \neg \phi | \phi \land \phi | \langle \pi \rangle \phi | K \phi \]
\[ \pi ::= a | ?\phi | \pi; \pi | \pi + \pi | \pi^* \]

- PDL language to specify complex plans and goals.
- Model checking problem as (constrained, conditional) contingent planning: \( M, s \models (a + b + ?Kp)^* (Kp \land \neg K[a]q) \).
- Different multi-agent extensions: in the same car or not?
Advantages of a (PDL-based) logical approach:

- various ways to do planning: model checking, sat checking, theorem proving (well studied in TCS)
- complicated goals: not just (positive) epistemic goals
- specification of plans with (epistemic) conditions and loops
- abstraction, refinement and equivalence of the plans
- in principle, probability can be plugged in
- to compare complexity of different planning problems as MC of fragments over various sub classes of models.
Thank you for your attention!
All That Is Gold Does Not Glitter by J. R. R. Tolkien

All that is gold does not glitter, 金子未必都发光，
Not all those who wander are lost; 游民未必是流氓。
The old that is strong does not wither, 老当益壮葆青春，
Deep roots are not reached by the frost. 根深蒂固经风霜。
From the ashes a fire shall be woken, 死灰复燃火势旺，
A light from the shadows shall spring, 昏天暗地光清扬。
Renewed shall be blade that was broken, 宝剑锋处断箝出，
The crownless again shall be king. 无冕之王又做庄！


Holliday, W., Hoshi, T., and Icard, T. (2012).
A uniform logic of information dynamics.

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Logics of public communications.
In Emrich, M. L., Pfeifer, M. S., Hadzikadic, M., and Ras, Z. W., editors, *Proceedings of the 4th International*
Symposium on Methodologies for Intelligent Systems, pages 201–216.


Wang, Y. and Li, Y. (2012). Not all those who wander are lost: dynamic epistemic reasoning in navigation.