Beyond “knowing that” Knowledge-how vs. Knowledge-that

Our formalization

FORMALIZING KNOWLEDGE-HOW

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Beyond “knowing that”

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Our formalization
A propositional modal logic that handles reasoning about knowledge (and belief) [von Wright 1951, Hintikka 1962]

- **Language**: “agent $i$ knows that $\phi$” ($K_i \phi$).
- **Model**: possibilities with *equivalence* relations.
- **Semantics**: you know that $\phi$ iff $\phi$ is true in all the situations that you consider *possible*.
- **Proof system**: §5

\[
p \land \neg K_i p
\]
Beyond “knowing that”: motivation

Knowledge is not only expressed in terms of “knowing that”:

- I *know whether* the claim is true.
- I *know what* your password is.
- I *know how* to go to Tsinghua.
- I *know who* proved this theorem.
- …

**Linguistically:** “know” takes embedded questions but “believe” does not; ambiguity in concealed questions.

**Philosophically:** are they reducible to “knowing that”?

**Logically:** how to reason about those forms of knowledge?

**Computationally:** how to efficiently represent and do inference about those knowledge expressions?
Beyond knowing that: difficulties and some results

New epistemic operators behave quite differently from the standard modal operator and are usually disguised FO-modal fellows, which causes difficulties for axiomatization and decidable machinery.

Some of our results:

- Knowing whether (non-contingency): axiomatizations and completeness proofs for its logic over various frame classes [Fan, Wang & van Ditmarsch: AiML14, RSL14 to appear]
- Knowing what: axiomatization and decidability for conditionally knowing what logic over FO epistemic models [Wang & Fan: IJCAI13, AiML14][Xiong 14][Ding 14]
- Knowing how: alternative non-possible-world semantics [Wang ICLA14]; philosophical discussion [Lau & Wang].
I know how to...

- swim
- ride a bike
- play piano
- go to Tsinghua
- prove Pythagoras theorem
Knowledge-how vs. Knowledge-that

- Intellectualism
- Anti-Intellectualism (especially Ability account)
Formalizing Intellectualism

Intellectualism: $\forall S \forall F \exists \varphi : KhSF \leftrightarrow KS\varphi$

Anti-Intellectualism: $\exists S \exists F \forall \varphi : (KS\varphi \land \neg KhSF) \lor (\neg KS\varphi \land KhSF)$
Anti-Intellectualism above

\[ \exists S \exists F \forall \varphi : (K_S \varphi \land \neg K_{h_S} F) \lor (\neg K_S \varphi \land K_{h_S} F) \] is necessarily false:

Given any \( S \) and \( F \):

- \( K_{h_S} F \) is either true or false
- if \( \varphi = \top \) then \( \neg K_{h_S} F \) (to make the disjunction true).
- if \( \varphi = \bot \) then \( K_{h_S} F \) (to make the disjunction true).
A revised formalization

The (bi-)implication should not be material:

$$\forall S \forall F \exists \varphi : \Box (Kh_S F \leftrightarrow K_S \varphi)$$

Correspondingly, the formulation of Anti-Intellectualism becomes:

$$\exists S \exists F \forall \varphi : \Diamond ((K_S \varphi \land \neg Kh_S F) \lor (\neg K_S \varphi \land Kh_S F))$$

To justify Intellectualism, we may give a truth condition of $Kh_S F$ in terms of knowledge-that, i.e., $K_S$-formulas.
A truth conditional approach

Stanley & Williamson (2001) defend Intellectualism against Ryle’s attack (1949), and proposed a truth condition for $Kh_S F$:

- There is a way $w$ for $S$ to $F$;
- $S$ knows that $w$ is a way for $S$ to $F$;
- $S$ entertains the proposition that $w$ is a way for $S$ to $F$, under *practical mode of presentation* (PMOP).

PMOP is linguistically motivated but not clearly justified. Also it is not in the form of knowledge-that.
Difficult questions

1. How to explain knowledge-how in comparative terms?

2. Do empirical findings really play a role in understanding knowledge-how?

3. How to exercise the corresponding knowledge-that to eventually do things?

4. Is the truth condition really free of implicit knowledge-how?

The ability account naturally avoids the problems above.
Johan is a basketball player who trains everyday to be a great 3-point shooter. At some point $t$, he actually gains the ability to score at least one third of the time when shooting 3-point field goals. However, he can only realize that he has this ability much later (after many more matches). At this point $t$, Johan would not say he knows how to score at least one third of the time, although he already has the corresponding ability.

Our goal is to reconcile Intellectualism and the ability account.
Our observations of KH

1. Knowledge-how often comes with implicit **preconditions** & the preconditions are propositional;

2. Both **goal** and **method** matter in knowledge-how. A goal can be viewed as a proposition, and a method is a collection of concrete ways, thus can be viewed as a predicate.

3. To know how to achieve a certain goal (a proposition) is to guarantee its truth by executing some way. The guaranteed truth is best formalised as a modal statement.
The $F$ in $Kh_S F$ is overloaded.

We claim every knowledge-how statement can be rephrased into the following extended form given its context:

\[
S \text{ knows how to achieve } \varphi \text{ by method } P \text{ under precondition } \psi
\]

We write it as: $Kh_S (\psi, P, \varphi)$.

We do not need to claim the converse to justify intellectualism.
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$Kh_S(\psi, P, \varphi)$ is further formalized by the following formula in a first-order modal language with modalities $K_S$, $\Box$, and $[w]_S$:

$$\exists w K_S (Pw \land \Box (\psi \rightarrow [w]_S \varphi))$$

where $[w]_S \varphi$ says that $S$ can guarantee $\varphi$ by executing $w$. As in the earlier discussion, $\Box$ is here to impose the strict implication from $\psi$ to $[w]_S \varphi$. 
Learning process of knowledge-how

Approaching to $\exists wK_s(\neg w \land \Box (\psi \rightarrow [w]_s \varphi))$:

- You know something is learnable:
  $K_s \exists w(\neg w \land \Box (\psi \rightarrow [w]_s \varphi))$ is true;

- After some training, you can realize the goal sometimes:
  $\neg v \land \Box (\psi \rightarrow \langle v \rangle_s \varphi)$ is true for some concrete way $v$;

- At some point, after enough training, you obtain the ability de facto: $\neg w \land \Box (\psi \rightarrow [w]_s \varphi)$ is true for some concrete $w$ which may be different from the previous $v$;
You finally obtain the knowledge-how by realizing that you have the ability: \( \exists wK_s(Pw \land \square(\psi \rightarrow [w]_s\varphi)) \) is true;

You may further improve your expertise by weakening the precondition \( \psi \), strengthening the postcondition \( \phi \), and tightening the method constraint \( P \). For example, to be a great 3-point shooter, Johan needs to raise his accuracy (\( \varphi \)), under stronger defense of the opponents (\( \psi \)) with a more stable and elegant posture (\( P \)).

Our slogan: *knowing your ability!*
Responses to the aforementioned problems

- Knowledge-how can be comparative: one’s knowledge-how may have a weaker precondition, a more restricted method, and a stronger goal when compared to others’ knowledge-how
The ‘procedural’ and ‘declarative’ distinction is about the representation and execution of the way \( w \), e.g., the representation of the way for swimming is distinctively different from the representation of the proof steps of Pythagoras theorem in cognitive science.

Empirical cognitive research may help us to see how different ways are represented, executed and learned in the brain.
According to our account, knowledge-how can be equivalently formulated as knowledge-that in the form of 
\[ \exists w K_S \theta(S, w, \psi, P, \varphi) \], where \( \theta \) is a proposition stating a fact about the ability of \( S \).

It does not make much sense to ask how to exercise the proposition \( \theta \) to get the corresponding ability or to do things. The real question to ask is how to execute the way \( w \), but as we discussed above, it is the very job of cognitive science.
Does $[w]_S \varphi$ always imply $S$ knows how to execute $w$ and lead to an infinite regression?

Knowing how to execute $w$ is not a necessary condition to $[w]_S \varphi$.

Moving fingers & the distinction between $K_S \exists w \theta$ and $\exists w K_S \theta$. 
Conclusion

- We propose a (formal) intellectualism and justify the truth condition approach.
- Knowledge-how can be roughly formalized by knowing that you have a certain ability.
- Having an ability is to guarantee a goal by some method under a precondition.
- It answers several difficult questions to Intellectualism.
- A logical language can clarify subtleties as our learning process shows.
Thank you!