Planning with epistemic goals

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1. Why does knowledge matter?

2. A mixed-blood single-agent framework

3. Multi-agent cases and further questions
Lost with a map at hand
Lost with a map at hand
Why does knowledge matter? A mixed-blood single-agent framework

Multi-agent cases and further questions
A rookie intelligence agent sneaking in an enemy building is guided by his headquarters. The communication with the HQ may be lost at some point. Now someone spotted him and pulled the alarm. In panic he got lost...

A mixed-blood single-agent framework

Multi-agent cases and further questions
Scenario in *Mission Impossible*

All of the following plans can *in fact* lead the agent to some safe place:

1. Moving right (*r*): the agent may not know that he is safe afterwards.
2. Moving up (*u*): the agent may know that he is safe afterwards, but he couldn’t know it beforehand.
3. Moving right and up (*ru*): agent knows that it will guarantee his safety before executing it.

Plans 1 and 2 are good if the planner is the HQ. Plan 3 is good for the agent as the planner.
What we want:

Uncertainty \(\xrightarrow{planning}\) Certainty (of the goal)

Knowledge and belief are notions that we use to organize pieces of information and to handle uncertainty (belief can be graded with probability).
Sources of uncertainty induce models of planning

- Initial states
- Non-deterministic actions
- Partial observability (higher-order in multi-agent cases)

<table>
<thead>
<tr>
<th>Uncertainty</th>
<th>init</th>
<th>obs</th>
<th>action</th>
<th>probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Classical</td>
<td>no</td>
<td>full</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>FOND</td>
<td>no</td>
<td>full</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>MDP</td>
<td>no</td>
<td>full</td>
<td>yes</td>
<td>yes</td>
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<tr>
<td>Conformant</td>
<td>yes</td>
<td>none</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>Contingent</td>
<td>yes</td>
<td>partial</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>POMDP</td>
<td>yes</td>
<td>partial</td>
<td>yes</td>
<td>yes</td>
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</tbody>
</table>
Planning with epistemic goals

The form of a plan:

- sequence of actions (no obs or too expensive)
- program/policy (some obs, conditional plan)

Planning: to reach some of the goal states under uncertainty (with the minimal expected cost).

- goals encoded by epistemic formulas (can be negative!)
- knowledge-based conditional actions
- make sure the planner knows that it will work!
Classical vs. epistemic

Roughly speaking:

- Classical planning: reachability over space of valuations.
- Epistemic planning: reachability over knowledge space.

There are translations to classical planning, e.g., [PG07].
Why epistemic logics?

Modality: mode of truth. Modal logics allow people separate truth from ‘truth’ under propositional attitudes: it is known that $p$, $p$ is permitted, $p$ is eventually true...

- Natural to express goals and conditions for actions
- Philosophically grounded: ‘sounds right’...
- Existing theoretical results and tools
- May handle a richer class of problems with similar cost

Not fully satisfied? We will come back to it at the end.
Before moving to the next part

Some warnings:

- Explicit transition models
- No probability
- No optimality on cost
- No heuristics
- Single-agent

However, we hope to capture the essences of logical aspects in epistemic planning.
Intelligence: think and move

Two modal logic approaches handling knowledge and actions:

- **Epistemic** Temporal Logic (ETL): knowledge in distributed systems based on temporal logic. [FHMV95, PR85]

- **Dynamic** Epistemic Logic (DEL): knowledge in multi-agent interactions based on epistemic logic. [Pla89, GG97, BMS98, vDvdHK07]
Two logical approaches about knowledge and action

<table>
<thead>
<tr>
<th>language</th>
<th>model</th>
<th>semantics</th>
</tr>
</thead>
<tbody>
<tr>
<td>ETL</td>
<td>time+K</td>
<td>temporal+epistemic</td>
</tr>
<tr>
<td>DEL</td>
<td>K+events</td>
<td>epistemic</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Kripke-like</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Kripke+d!namic</td>
</tr>
</tbody>
</table>

\[ \neg Kp \land F Kp \]

\[ \neg Kp \land [!p]Kp \]

Dynamic semantics: the meaning of an event is the *change* it brings to the knowledge states. (dates back to Stalnaker).
A mixed-blood baby of DEL and ETL [WL12]

We may not construct the temporal structure from scratch.

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<td>epistemic</td>
</tr>
<tr>
<td>Mixed</td>
<td>K+events</td>
<td>temporal+epistemic</td>
</tr>
</tbody>
</table>

Such a logic usually cannot be reduced to epistemic logic, but techniques developed in [WC13, WA13] give a general method to axiomatize DEL-like logics without reductions.
Model: Kripke model with an uncertainty set

Given a countable set $P$ of basic propositions and a finite set $A$ of basic actions:

- A Kripke model: $\mathcal{N} = \langle S, \{R_a \mid a \in A\}, V \rangle$
  where $S \neq \emptyset$, $R_a \subseteq S \times S$ and $V : P \to \mathcal{P}(S)$.
- An uncertainty map (UM):
  $\mathcal{M} = \langle S, \{R_a \mid a \in A\}, V, U \rangle$
  where $U \neq \emptyset$, $U \subseteq S$ such that $\forall s, t \in S$, $e(s) = e(t)$.
- $\mathcal{M}, s$ is a pointed UM model, if $s \in U$.

Example ($\mathcal{M}, s_3$)

```
S7 ← l ← S6
   ↑   ↑   ↑   ↑
S8: Safe  S9: Safe
S1 ← r ← S2 ← r ← S3 ← r ← S4: Safe ← r ← S5
```

\[ \]
Epistemic Action Language

- **EAL language with action and knowledge as modalities:**

\[
\phi ::= \top \mid p \mid \neg \phi \mid (\phi \land \phi) \mid \langle a \rangle \phi \mid K \phi
\]

where \( p \in P, \ a \in A.\)

- For abbreviations: \( \bot := \neg \top, \ \phi \lor \psi := \neg (\neg \phi \land \neg \psi), \ \phi \rightarrow \psi := \neg \phi \lor \psi, [a] \phi := \neg \langle a \rangle \neg \phi, \ \hat{K} \phi := \neg K \neg \phi.\)

- The intuition of the formula:
  - \( K \phi \) says that the agent knows that \( \phi \)
  - \( \langle a \rangle \phi \) says that after doing \( a \) it is possible that \( \phi \) holds.
Semantics of EAL on UM: actions as changes

Given any UM model \( \mathcal{M} = \langle S, \{R_a \mid a \in A\}, V, U \rangle \), the satisfaction relation on pointed UM model \( \mathcal{M}, s \) is defined as:

\[
\mathcal{M}, s \models K\phi \iff \forall u \in U : \mathcal{M}, u \models \phi
\]
\[
\mathcal{M}, s \models \langle a \rangle\phi \iff \exists t \in S \text{ such that } s \xrightarrow{a} t \text{ and } \mathcal{M}|^a_t, t \models \phi
\]

- \( \mathcal{M}|^a_t = \langle S, \{R_a \mid a \in A\}, V, U|^a_t \rangle \)
- \( U|^a_t = U|^a \cap E(t) \) ‘carry’ the circle along \( \xrightarrow{a} \), and then check with the observations at the current location
- \( U|^a = \{r' \mid \exists r \in U \text{ such that } r \xrightarrow{a} r' \} \)
- \( E(t) = \{t' \mid e(t') = e(t)\} \)

The observational power can be more abstract.
Why does knowledge matter? A mixed-blood single-agent framework

Example

\[ M, s_1 \models K \neg p \land \langle b \rangle \neg Kp \] and \[ M|_{s_3}^b, s_3 \models \neg Kp \]

Example (path dependency)

\[ M, s_1 \models K \neg p \land \langle a \rangle \langle a \rangle Kp \] and \[ (M|_{s_2}^a)|_{s_3}^a, s_3 \models Kp \]
Features of the semantics

- Truth value of EAL formulas are *not* defined on every state in a model.
- Knowledge is ‘path-dependent’.
- $U$ is an epistemic snapshot of an unravelled ETL-like model.
- We say a formula $\phi$ is valid ($\models \phi$), if for any pointed UM model $\mathcal{M}, s$: $\mathcal{M}, s \models \phi$. 
The scenario in *Mission Impossible*

\[ M, s_3 \models [r](\text{Safe} \land \neg \text{KSafe}) \]
(HQ guides you safe but you do not know it)

\[ M, s_3 \models [u](\text{Safe} \land \text{KSafe}) \]
(HQ guides you safe and you know it)

\[ M, s_3 \models K[r][u]\text{Safe} \]
(You know the plan will guarantee your safety)
Theoretical aspects

- Normal form: $K$ can be pushed outside $\langle a \rangle$
- A bisimulation notion
- Finite model property
- A sound and complete axiomatization
A sound and complete axiomatization $S_{\text{EAL}}$

Rules:
- MP
- NECK
- NEC$(a)$
- SUB

Axioms:
- TAUT: all the axioms of propositional logic
- DISTK: $K(p \to q) \to (Kp \to Kq)$
- DIST$(a)$: $[a](p \to q) \to ([a]p \to [a]q)$
- T: $Kp \to p$
- 4: $Kp \to KKp$
- 5: $\neg Kp \to K\neg Kp$
- OBS$(a)$: $K\langle a \rangle \top \lor K\neg \langle a \rangle \top$
- PR$(a)$: $K[a]p \to [a]Kp$
- NL$(a)$: $\land_{B \subseteq A}(\langle a \rangle(\psi_B \land Kp) \to K[a](\psi_B \to p))$

where $\psi_B$ is describes the availability of the actions.

Why do we care about complete axiomatization (semantically valid formulas are syntactically generated by these axioms and rules)?
A short summary

- Model: compact (Kripke models with circles)
- Language: simple ($K$ and $\langle a \rangle$)
- Semantics: dynamic and epistemic
- Use: knowledge tracking and plan verification
Conformant planning

Given the initial uncertainty, given that no feedback is available during the execution, find a sequence of available (non-deterministic) actions that will always guarantee the goal. ‘No matter where you are, call the taxi to go to IMSc!’

Definition (Simplified semantics without observations)

Given any uncertainty map $\mathcal{M} = \langle S, \{ R_a \mid a \in A \}, \mathcal{V}, U \rangle$ and any point $s \in U$, the simplified semantics is defined as follows:

$\mathcal{M}, s \models \langle a \rangle \phi \iff \exists t \in S : s \xrightarrow{a} t$ and $\mathcal{M}|^a, t \models \phi$

$\mathcal{M}, s \models K \phi \iff \forall u \in U : \mathcal{M}, u \models \phi$

where $\mathcal{M}|^a = \langle S, \{ R_a \mid a \in A \}, \mathcal{V}, U|^a \rangle$ and $U|^a = \{ r' \mid \exists r \in U$ such that $r \xrightarrow{a} r' \}$. For a sequence of actions $\sigma = a_1 \ldots a_n$, we write $U|^{\sigma}$ for $(\ldots ((U|^{a_1})^{a_2}) \ldots )^{a_n}$. and write $\mathcal{M}|^{\sigma}$ for $(\ldots (((\mathcal{M}|^{a_1})^{a_2}) \ldots )^{a_n}$. 

Let $\langle a \rangle \phi$ be the shorthand of $[a] \phi \land \langle a \rangle \phi$.

**Definition (Conformant planning)**

Given an uncertainty map $\mathcal{M}$, a goal formula $\phi$, and a set $B \subseteq A$, the conformant planning problem is to find a finite (possibly empty) sequence $\sigma = a_1 a_2 \cdots a_n \in \mathcal{L}(B^*)$ such that for each $u \in U_\mathcal{M}$ we have $\mathcal{M}, u \models \langle a_1 \rangle \langle a_2 \rangle \cdots \langle a_n \rangle \phi$. The existence problem of conformant planning is to test whether such a sequence exists.
Intuitively, we want a plan which will never fail w.r.t. non-deterministic actions and initial uncertainty of the agent. E.g., \( ru \) is a conformant plan to the agent.

\[
\begin{align*}
S_7 & \leftarrow l \rightarrow S_6 \\
& \uparrow \quad \uparrow \\
& \quad u \\
S_1 & \rightarrow r \rightarrow S_2 \rightarrow r \rightarrow S_3 \rightarrow r \rightarrow S_4 \rightarrow r \rightarrow S_5 \\
& \quad \uparrow \quad \uparrow \quad \uparrow \\
& \quad \quad u \\
S_8 & \text{: Safe} \\
S_9 & \text{: Safe}
\end{align*}
\]

For all \( u \in U \mathcal{M} \), \( u \models (r) (u) KSafe \quad \iff \quad \mathcal{M}, s_3 \models K (r) (u) KSafe \)

We can verify conformant plans by model checking EAL. What about checking the existence of a plan?
To express the existence of an unbound conformant plan

**Enriched language EPDL**

\[
\phi ::= \top \mid p \mid \neg \phi \mid \phi \land \phi \mid [\pi] \phi \mid K \phi
\]

\[
\pi ::= a \mid ?\phi \mid \pi; \pi \mid \pi \cup \pi \mid \pi^*
\]

E.g., EPDL can express \(K[(?\neg Kp; a)^*; ?Kp; b]K[c]q\)

<table>
<thead>
<tr>
<th>(\mathcal{M}, s \models [\pi] \phi)</th>
<th>(\iff)</th>
<th>for all (\mathcal{M}', s' : (\mathcal{M}, s)[[\pi]](\mathcal{M}', s')) implies (\mathcal{M}', s' \models \phi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>((\mathcal{M}, s)[A](\mathcal{M}', s'))</td>
<td>(\iff)</td>
<td>(\mathcal{M}' = \mathcal{M}</td>
</tr>
<tr>
<td>((\mathcal{M}, s)[?] \psi](\mathcal{M}', s'))</td>
<td>(\iff)</td>
<td>((\mathcal{M}', s') = (\mathcal{M}, s)) and (\mathcal{M}, s \models \psi)</td>
</tr>
<tr>
<td>((\mathcal{M}, s)[\pi_1; \pi_2](\mathcal{M}', s'))</td>
<td>(\iff)</td>
<td>((\mathcal{M}, s)[\pi_1] \circ [\pi_2](\mathcal{M}', s'))</td>
</tr>
<tr>
<td>((\mathcal{M}, s)[\pi_1 + \pi_2](\mathcal{M}', s'))</td>
<td>(\iff)</td>
<td>((\mathcal{M}, s)[\pi_1] \cup [\pi_2](\mathcal{M}', s'))</td>
</tr>
<tr>
<td>((\mathcal{M}, s)[\pi^*](\mathcal{M}', s'))</td>
<td>(\iff)</td>
<td>((\mathcal{M}, s)[\pi]^*(\mathcal{M}', s'))</td>
</tr>
</tbody>
</table>
Recall: a conformant plan requires that for each \( u \in U_M \) we have \( \mathcal{M}, u \models \langle a_1 \rangle \langle a_2 \rangle \cdots \langle a_n \rangle \phi \).

**Proposition**

There exists a conformant plan w.r.t. \( B \subseteq A \) and \( \phi \in \text{EPDL} \) on \( \mathcal{M}, s \) iff \( \mathcal{M}, s \models \langle (\sum_{a \in B} (?K \langle a \rangle \top; a))^* \rangle K \phi \).

Call the formula \( \theta_{B,\phi} \). If \( B = \{ a_1, a_2 \} \) then

\[
\theta_{B,\phi} = \langle ((?K \langle a_1 \rangle \top; a_1) + (?K \langle a_2 \rangle \top; a_2))^* \rangle K \phi.
\]

Intuitively, the conformant plan consists of actions that are always executable given the uncertainty of the agent (guaranteed by the guard \( K \langle a \rangle \top \)). In the end the plan should also make sure that \( \phi \) must hold given the uncertainty of the agent (guaranteed by \( K \phi \)).
Simple-minded solution does not work

What about $K(\langle \Sigma B \rangle^* \phi)$?

**Example**

Let $U = \{s_1, s_2\}$, uncertainty map $M = \langle N, U \rangle$ and the goal formula is $p$. Let $B = \{a, b\}$, we have $M, s_1 \models K(\langle \Sigma B \rangle^* p$, but there is no solution to this conformant planning problem.

```

[Diagram: s_1 \xrightarrow{a} s_3 \xrightarrow{b} s_5 : p
 s_1 \xrightarrow{b} s_4 \xrightarrow{a} s_6 : p]
```
The last $K$ is important

What about $\langle (\Sigma_{a \in B}(?K \langle a \rangle \top; a))^* \rangle \phi$?

Example

Let $U = \{s_1\}$, and let the goal formula be $p$. As we can see, there is no solution to this conformant planning problem. Indeed $\mathcal{M}, s_1 \not\models \langle (\Sigma_{a \in B}(?K \langle a \rangle \top; a))^* \rangle Kp$ with $B = \{a, b\}$, but we could have $\mathcal{M}, s_1 \models \langle (\Sigma_{a \in B}(?K \langle a \rangle \top; a))^* \rangle p$.

\[
\begin{align*}
\begin{array}{ccc}
{s_1} & \xrightarrow{a} & s_2 \\
 & \xrightarrow{b} & s_5 : p \\
 & & \downarrow b \\
& & s_4
\end{array}
\end{align*}
\]
Model checking EPDL (ongoing work)

Usual global model checking algorithm is not applicable since the semantics of $K$ is path dependent. The following equivalent semantics suggests a (local) recursive MC algorithm for star-free EPDL.

Definition (Alternative semantics without model changing)

Given any uncertainty map $\mathcal{M}$ and any point $s \in U$, the semantics is defined as follows w.r.t. a sequence of actions $\sigma$:

\[
\begin{align*}
\mathcal{M}, s \models \phi & \iff \mathcal{M}, s \models \epsilon \phi \\
\mathcal{M}, s \models \sigma T & \iff \text{always} \\
\mathcal{M}, s \models \sigma p & \iff p \in \mathcal{V}(s) \\
\mathcal{M}, s \models \sigma \neg \phi & \iff \mathcal{M}, s \not\models \sigma \phi \\
\mathcal{M}, s \models \sigma (\phi \land \psi) & \iff \mathcal{M}, s \models \sigma \phi \text{ and } \mathcal{M}, s \models \sigma \psi \\
\mathcal{M}, s \models \sigma K \phi & \iff \text{for all } v \in U|\sigma : \mathcal{M}, v \models \sigma \phi \\
\mathcal{M}, s \models \sigma (\pi) \phi & \iff \text{there is a } \omega \text{ in } \mathcal{L}(\pi) \text{ such that } s \xrightarrow{\omega} t \\
\text{for some } t \text{ and } \mathcal{M}, t \models \sigma r(\omega) \phi
\end{align*}
\]
Advantages of such a logical approach

- complicated goals
- see subtleties
- specification and verification of plans with (epistemic) conditions and loops
- abstraction, refinement and equivalence of the plans
- (in principle) probability may be plugged in
- to compare complexity of different planning problems as MC of fragments over various sub classes of models.

To extend the current framework to the multi-agent setting, we need another core idea of DEL.
Multi-agent planning

It makes much sense even without epistemics:

- Classical planning for homogeneous/heterogeneous agents: diamond thieves, another hand...
- It is important to have a better understanding of the ‘necessary cooperation’.
- Not much work has been done.
Goal formula may involve higher-order knowledge, which requires the observability to be higher-order too.

The best way to capture higher-order observability so far is action models in DEL (Kripke-model-like).

- states: pointed epistemic models
- actions: finite set of action models (with factual change)
- state transition function: update product $\otimes$. 
Example

How to let agent $i$ know a secret number and let others merely know that $i$ knows but they do not know the number?

![Diagram](image)

Full observability, partial observability, and non-observable.
(Un)decidability results (S5, K45, no common knowledge, no factual changes)

The undecidable cases for (unbound) plan existence problem:
- Multi-agent [BA11, AB13]

Decidable fragments:
- Agent restriction: single-agent [AB13]
- Action model restriction: multi-agent propositional action models [LPW11, YWL13]
Other related epistemic logical approaches

- ETL-like model with trials [NR09]
- Planning with knowledge-based programs, [LZ12]: single-agent, see [Jen13] for connection with DEL
- Conditional DEL plans [ABJ12]: single-agent
- DEL-like sequents [Auc12]: one-step, no class of action models
- Model checking with Alternating Temporal Epistemic Logic [vdHW02]: finite ATEL model
- Multi-agent situation calculus with action (plausibility) model [FL13]
- Recent theses of Bolander’s PhD students
- Van Ditmarsch’s work on card protocols
Multi-agent EP: questions

- Who is the planner?
- Executability of action models: observation and actions
- Local actions and decomposition of action models
- Action language and domain language
- Communication and concurrency
- Conflicting goals
- Applications of higher-order epistemic planning?

Long way to go? That is your chance!
A mathematician, a computer scientist and a philosopher walked into a bar.
Logicians: self-image and reality

From Sicun Gao:

Logicians want to be:

- Mathematicians of mathematicians
- Computer scientists of computer scientists
- Philosophers of philosophers

However, they often end up being:

- Mathematicians to philosophers
- Computer scientists to mathematicians
- Philosophers to computer scientists
MPC United!

- Mathematics: Rigorous
- Computer science: Implementable
- Philosophy: To the point

To ‘practising’ computer scientists: you may use logic as a guide who leads you to philosophically sound ideas with mathematical rigorous proofs of what you can/cannot do.

Then bear it in mind and seek for the most efficient way that works in practice.
Thank you for your attention!
Why does knowledge matter? A mixed-blood single-agent framework Multi-agent cases and further questions
PSPACE lower bound

How hard is model checking EPDL on uncertainty maps?

Given a formula $\alpha$ in the shape $Q_1 x_1 Q_2 x_2 \ldots Q_n x_n \phi(x_1, \ldots, x_n)$ where $Q_i$ is $\exists$ if $i$ is odd and $Q_i$ is $\forall$ if $i$ is even and $\phi$ is a CNF based on variables $x_1, \ldots, x_n$.

The validity checking of $\alpha$ can be reduced to model checking

$$\langle (a_1 + \bar{a}_1); ?(p_1 \lor q_1) \rangle [ (a_2 + \bar{a}_2); ?(p_2 \lor q_2) ] \cdots \psi(p_1, \cdots, p_n, q_1, \cdots, q_n)$$

where $\psi(p_1, \cdots, p_n, q_1, \cdots, q_n)$ is obtained from $\phi(x_1, \ldots, x_n)$ by replacing each $x_i$ with $\hat{K} p_i$ and replacing each $\neg x_i$ with $\hat{K} q_i$. 
Guillaume Aucher and Thomas Bolander.
Undecidability in epistemic planning.

Mikkel Birkegaard Andersen, Thomas Bolander, and Martin Holm Jensen.
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Del-sequents for regression and epistemic planning.

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Epistemic planning for single and multi-agent systems.
A. Baltag, L. Moss, and S Solecki.
The logic of public announcements, common knowledge, and private suspicions.

R. Fagin, J. Halpern, Y. Moses, and M. Vardi.
Reasoning about knowledge.

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Multiagent knowledge and belief change in the situation calculus.

J. Gerbrandy and W. Groeneveld.
Reasoning about information change.
Why does knowledge matter? A mixed-blood single-agent framework

Multi-agent cases and further questions


Martin Holm Jensen.
Planning using dynamic epistemic logic: Correspondence and complexity.

Benedikt Löwe, Eric Pacuit, and Andreas Witzel.
DEL planning and some tractable cases.

Jérôme Lang and Bruno Zanuttini.
Knowledge-based programs as plans - the complexity of plan verification.

**Rajdeep Niyogi and Ramaswamy Ramanujam.**
An epistemic logic for planning with trials.

**Héctor Palacios and Hector Geffner.**
From conformant into classical planning: Efficient translations that may be complete too.

**J. A. Plaza.**
Logics of public communications.

R. Parikh and R. Ramanujam.
Distributed processes and the logic of knowledge.

Wiebe van der Hoek and Michael Wooldridge.
Tractable multiagent planning for epistemic goals.

H. van Ditmarsch, W. van der Hoek, and B. Kooi.
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An alternative axiomatization of DEL and its applications.  

Yanjing Wang and Qinxiang Cao.  
On axiomatizations of public announcement logic.  

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Multi-agent epistemic explanatory diagnosis via reasoning about actions.