Paradox of Deniability

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Disagreement

- Suppose Lisa disagrees with Marc as to whether Padua is North of Venice.
- Marc asserts that it is: Padua is North of Venice.
- Lisa disagrees.
- Lisa may express this by asserting the negation of what Marc said: Padua is not North of Venice.
- Whatever the exact details of one's account of disagreement, a way for a speaker–Lisa–to disagree (with Marc) is to negate what the other is asserting.

Disagreement: a formulation

- In general we can argue that two speakers disagree only if they have incompatible beliefs, or perform speech acts that cannot be jointly correct:

> *Disagreement. Two speakers S_1 and S_2 who respectively utter u_1 and u_2 disagree only if u_1 and u_2 cannot both be correct.*

And a standard way to express that one has an incompatible belief with another is to negate the other's assertion as for Lisa and Marc.

Disagreement from a dialetheic perspective

- Question: Can Lisa be a dialetheist – one who thinks there are dialetheiae, i.e. sentence both true and false?
- No, she can't! And dialetheists may not in general follow suit.

Disagreement from a dialetheic perspective

- if A is a dialetheia (or a *glut*), i.e. if it is both true and false, dialetheists won't in general take assertions of both A and $\neg A$ to express disagreement.
- And, in the same vein, if A is a dialetheia, they may well express agreement by asserting $\neg A$ in response to the assertion that A.
- This directly follows from their conception of negation.

Exclusive and non-exclusive negation. Negation in a dialetheic perspective in a nuthshell

- Boolean or *exclusive negation* is a propositional connective $\neg$, such that, by virtue of its very meaning, α and $\neg\alpha$ are *incompatible*.
- In other words, α and $\neg\alpha$ cannot be both true; i.e., it is excluded that α and $\neg\alpha$ are both true.
- Priest's dialetheism argues for its nonexistence (for example, see Priest 1993, 1998, 2006a, 2006b).

Negation in a dialetheic perspective in a nuthshell

- If α is a dialetheia, they may well express agreement by asserting $\neg\alpha$ in response to the assertion that α.
- Question: How to express disagreement in a dialetheic framework?

Disagreement and dialetheism

- Disagreement may still be expressed by denying what was said (Priest 2006, Priest 2006a, Beall 2009).
- Lisa may express her disagreement by denying that Padua is North of Venice.
- How to follow this suggestion?

Negation in a dialetheic perspective in a nuthshell

- To make up for the lack of exclusive negation, Priest introduces the notion of denial of a sentence or rejection (here we use the two terms indifferently), understood as a *speech act*, as clearly distinguished from the acceptance of the negation of α (Priest 1993, 1998, 2006a, b).
- Following Priest (2006), I take assertion and denial or rejection to be external manifestations of, respectively, the mental states of acceptance and rejection.
- Thesis: Assertion and rejection are incompatible speech acts.

Denial and Disagreement

- In particular, in order for this to work, the assertion of $\neg A$ must not commit one to denying A: denial must be a primitive speech act, not reducible to the assertion of $\neg A$ (Parsons 1984, Priest 2006).

- That is, dialetheists must reject, and reject, the right-to-left direction of the classical theory of denial, that to deny A is equivalent to asserting $\neg A$:

One correctly denies A if and only if one correctly asserts $\neg A$.

Denial and Disagreement

- Unlike paraconsistent negation, denial is for dialetheists exclusive: A and $\neg A$ may both be true, but you can't correctly assert and deny A. As said: Assertion and rejection are incompatible speech acts.
- The exclusivity of negation is lost (or perhaps was never had) in the realm of logic, but it is regained at the pragmatic level – with denial.
- The above characterization in terms of incompatibility seems to go in the direction before expressed by disagreement.

the Question of the talk

Question: If giving up the exclusivity of negation is the key to solving the semantic paradoxes, doesn't the exclusivity of denial land us back in paradox?

Our plan today

- I argue, first in an informal way, then I sketch it formally, that intuitive norms for denial (which dialetheists arguably accept) give rise to paradox – the rejectability paradox – provided one's language is rich enough to express deniability.

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Our plan today

- I argue, first in an informal way, then I sketch it formally, that intuitive norms for denial (which dialetheists arguably accept) give rise to paradox – the rejectability paradox – provided one's language is rich enough to express deniability.
- Then, I consider some possible ways out or replies, concerning first the informal way of characterizing the paradox, then the formal language adopted.
- I conclude with a dilemma: either denial can serve as means to express disagreement, but the notion of exclusive deniability is not expressible in the glut theorist's language, or deniability is expressible, but denial no longer is a means to express disagreement.

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On the assertaibility (or deniability) liar

- As Littman and Simmons (2004) observe since the dialetheist appeals to 'non-standard relations between assertability and deniability', we are owed a full account of these notions.
- Any such account would need to deal with apparent paradoxes that turn on the notion of assertability or deniability.

On the assertability liar

- Littman and Simmons (2004) have introduced a paradox called the assertability paradox. Take sentence α as having the form
- (α) α is not assertable.
- They argue that (α) is a dialetheia.

On the assertability liar

- Suppose (α) is true. Then what it says is the case. So (α) (i.e. (α) α is not assertable) is not assertable. But we have just asserted (α). So (α) is assertable–and we have a contradiction.
- Suppose, on the other hand, that (α) is false. Then what (α) says is not the case. So (α) is assertable. So we may assert: (α) is not assertable. Again, we have a contradiction.

(Littman and Simmon 2004, 320)

On the assertability liar. A mistake

Before discussing why the assertability paradox should be a problem for a dialetheist, observe that the argument is not correct in the above formulation:

- The mere supposition that α is true does not imply its assertability. Indeed, assertability implies the recognition, not just the mere supposition, of the truth of (α).

On the amended asserability liar

To amend the argument, let us prove dialetheically that α is true by distinguishing the following two cases:

- (1) Assume that (α) is false. Then, its negation is true, so (α) is assertable and then it is true.
- (2) Assume that (α) is true. Then it is true.
- According to the Law of the Excluded Middle (LEM), (α) is true. In this way we have a proof–not just a supposition–of the truth of (α).
- And we can assert it. So (α) is assertable, in opposition to what (α) claims (it claims of being not assertable), hence it is false. Therefore it is a dialetheia.

On the assertabibility paradox

- If α is a dialetheia, then it both is and is not assertable. But how is it possible to both assert and not assert a sentence?
- This seems to be impossible also for a dialetheist.
- It seems that while acknowledging that certain sentences can be both true and false, you cannot admit that a sentence is assertable and not assertable.

On the assertability paradox

- Is the above conclusion a real problem for a dialetheist? The quick reply is, $\surd$ *No* $\surd$.
- Once the exclusivity of logical negation has been rejected, the non-assertability of a sentence does not exclude its assertability.
- Take for example the above exemplified case of agreement of Marc and Lisa.
- Even if it is far from clear what it means that a certain sentence is and is not assertable!

On the rejectability (or deniability) paradox

- Let R be a sentence having the form:
- (R) the sentence R is rejectable.

On the rejectability paradox

- Assuming that (R) is true, then it is rejectable. So, there is a state of knowledge in which one can reject it. In such a state, one recognizes that what (R) says is true, so that one is in a position to assert (R). So you both reject and accept R.

- Thus, the assumption of (R) leads to a state (of knowledge) in which one can both assert and reject (R), and that is dialetheically inacceptable.

- It follows that (R) cannot be true. But then we can reject it, recognize its truth and assert it, which, again, is in opposition to Priest's thesis of the impossibility of accepting and rejecting the same sentence. Again: dialetheically inacceptable.

On the rejectability paradox. A reply

Priest does not analyse the rejectability paradox.

However, he discusses a similar paradox called the irrationalist paradox in Priest (2006a, 111–112 and 2010 121). Let I be the sentence saying that itself is not rationally acceptable (believable):

- (I): it is not rational to accept I

On the rejectability paradox. A dialetheist reply

- (I): it is not rational to accept I

Suppose that one accepts it; then one accepts I and *it is not rational to accept* I. This, presumably, is irrational. Hence it is not rational to accept I. That is, we have just proved I. So it is rational to accept it.

On the rejectability paradox. A dialetheist reply

Priest's derivation goes roughly as follows (Priest 2010, 120). Let Rat be an operator expressing *rational acceptance* and $R = \neg \text{Rat}(R)$, Priest derives R from the schema

$$(P)\ \neg \text{Rat}(A \wedge \neg \text{Rat}(A)),$$

as follows:

$$\cfrac{\cfrac{\cfrac{\neg \text{Rat}(R \wedge \neg \text{Rat}(R))}{\neg \text{Rat}(R \wedge R)}}{\neg \text{Rat}(R)}}{R}\ .$$

On the rejectability paradox. A dialetheist reply

$$\cfrac{\cfrac{\cfrac{\neg\mathrm{Rat}(R \wedge \neg\mathrm{Rat}(R))}{\neg\mathrm{Rat}(R \wedge R)}}{\neg\mathrm{Rat}(R)}}{R}\,.$$

That is, R, and hence $\neg\mathrm{Rat}(R)$, is deducible from P: $P \vdash R$ (remember: $R = \neg\mathrm{Rat}(R)$).

Assuming that rational acceptance Rat is closed under single-premise deducibility, and that P is rationally acceptable (*and it seems to be: if someone believes A, and, at the same time, believes that it is not rationally permissible to believe A, that would seem to be pretty irrational – not something that is itself rationally permissible*), we have that $\mathrm{Rat}(P) \vdash \mathrm{Rat}(R)$, and we have a contradiction.

On the rejectability liar. A dialetheist reply

Priest calls such kind of paradox a rational dilemma.

He observes that a dialetheist cannot rule out a priori the occurrence of rational dilemmas:

> *Arguably, the existence of dilemmas is simply a fact of life (Priest 2006b, 111).*

On the rejectability liar. A dialetheist reply

- Moreover, he maintains that the irrationalist's paradox is much more problematic for a classicist than for a dialetheist.
- For the latter it is not irrational, Priest argues, to believe both a sentence α and that it is irrational to believe α, if such a belief is also rational, an option clearly closed to a fan of classical logic.
- This argument is in line with our above comment to the assertability paradox: if negation is non-exclusive, a dialetheist can rightly assert a non-assertable sentence, if she has recognized that it is also true.

First reply. The irrationalist paradox with rejection

- (I): it is not rational to believe I

I is asseratble and rejectale

- Suppose that a rational human being believes I. Then she should accept to have an irrational belief, against her rationality. So she can rightly reject it.
- But, then, she recognize that it is irrational to believe it, namely that I is true. Therefore, we are right to assert it.

Thus, as in the case of the rejectability paradox, we have good reasons for asserting, as well as good reasons for rejecting, the same sentence.

On the rejectability liar. A reply to the reply

Observe that:

- both the irrationalist's paradox with rejection and the rejectability paradox, lead to the conclusion that it is rational both to assert and reject it, against Priest's thesis that assertion and rejection are incompatible speech acts.
- On the other hand, the abandon of that thesis would destroy the attempt of recovering the notion of exclusivity by transferring it from negation to rejection.

On the rejectability liar. A first reply to the reply

About rational dilemma:

- It it a solution to the paradox? No.
- Consider a classicist and the strengthened liar. He could observe that the fact that there are rational grounds both for the truth and for the untruth of the strengthened liar sentence is nothing but a rational dilemma. It is simply a fact of life. There is nothing to add.
- But then the strengthened liar does not suggest the presence of dialetheias. The semantic paradoxes have nothing to do with the alleged existence of dialetheias!

On the rejectability paradox

- That is the rejectability paradox in an informal way. And why it could be problematic for a dialetheist.
- Problem: let us seriously consider the dialetheist analysis of assertion and denial. Is possible to provide a logic of exclusive denial for glut-theorist?
- In the next I introduce a bilateral extension of Priest's Logic of Paradox LP to include denial.

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Dialetheist theories

- We'll assume (with our target theorists) that the truth predicate at least satisfies the T-Scheme

$$(\textit{T-Scheme})\ Tr(\ulcorner A \urcorner) \leftrightarrow A.$$

- The logic is Priest's Logic of Paradox LP, which may or may not be augmented with a suitable conditional $\rightarrow$ satisfying $A \rightarrow A$ and *modus ponens*.

LP

- Semantically, LP is a three-valued logic whose language is that of CPL and admissible valuations are all the total maps from the set of well-formed formulae *WFF* to the set $\{1, 0.5, 0\}$ satisfying:

  ($\neg$) $v(\neg A) = 1 - v(A)$
  ($\vee$) $v(A \vee B) = \max\{v(A), v(B)\}$
  ($\wedge$) $v(A \wedge B) = \min\{v(A), v(B)\}$

- The designated values are $\{1, 0.5\}$: an argument is LP-valid if it never gets you from either 1 or 0.5 to 0.
- Notable invalidities include the principle of Ex Contraditione Quodlibet

$$\text{(ECQ) } A, \neg A \vdash B.$$

Introducing denial

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- How to introduce denial in a dialetheic framework?
- As said, following Priest 2006, we take assertion and denial to be external manifestations of, respectively, the mental states of acceptance and rejection.
- One way is to represent them by means of signed formulae: $+A$ for assertion and $-A$ for denial, where $+$ and $-$ are nonembeddable force signs (Rumfitt 2000).

Introducing denial

- Following Smiley (1996), one may interpret $+A$ and $-A$ as yes-or-no questions, respectively reading *A? Yes!* and *A? No!*.
- Formalizations of classical logic in which rules are given for asserting and denying complex propositions are in (Smiley 1996, Rumfitt 2000, Humberstone 2000).

Introducing denial

They essentially comprise:

(i) what Rumfitt calls signed positive *analogues* of the standard rules for intuitionistic logic, i.e. standard rules such as ∨-I, written in a yes-or-no format

$$+\text{-}\vee\text{-I}\ \frac{\Gamma \vdash +A}{\Gamma \vdash +(A \vee B)}\ ,$$

Introducing denial

and (ii) what Rumfitt calls *coordination principles*: structural rules governing the logic of $+$ and $-$, such as the following:

$$\text{SR}\ \frac{\Gamma, +A \vdash +B \qquad \Delta, +A \vdash -B}{\Gamma, \Delta \vdash -A}\ .$$

Classical denial

- Classically, one defines a set of correctness-valuations C for signed formulae such that every member is induced by the set of admissible truth-valuations of CPL by the following correctness clauses:

(C1) $v_c(+A) = \mathbf{1}$ iff $v(A) = 1$;
(C2) $v_c(-A) = \mathbf{1}$ iff $v(A) = 0$.

- One may correctly assert (deny) A just in case A is true (false).
- Validity for signed formulae is defined in the obvious way: an argument is valid iff it preserves value $\mathbf{1}$, i.e. correctness.
- This won't work in a dialetheist framework, however.

Dialetheist assertion and denial

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- In short: you may assert A iff A is true (and possibly false, too), and you may deny A iff A is false (even if it turns out to be also true).

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- The natural dialetheist counterparts of C1 and C2 are:

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- In short: you may assert A iff A is true (and possibly false, too), and you may deny A iff A is false (even if it turns out to be also true).
- But, if one could correctly deny what may be correctly asserted – a straightforward consequence of C1*, C2* and the existence of gluts – the denial of A would no longer be guaranteed to express disagreement.

Dialetheist assertion and denial

This is why Lisa's assertion may not in general express disagreement with Marc's assertion: in a glut-theoretic framework, in the way above specified, utterances may *both* be correct.

Dialetheist assertion and denial

If denial/rejection is a means of expressing disagreement, denial must be exclusive: it must be "impossible jointly to accept and reject the same thing' (Priest 2006a, p. 103).

But, then, C2 must *not* be revised (with C2*): the exclusive denial of A is correct iff A is (classically) *false only*, i.e. false but not true.

(C2) $v_c(-A) = \mathbf{1}$ iff $v(A) = 0$.

(C2*) $v_c(-A) = \mathbf{1}$ iff $v(A) \subseteq \{0, 0.5\}$

LP*

- We thus get a *logic of exclusive denial*, call it LP*, whose language is the language of LP supplemented with signs expressing assertion ($+$) and denial ($-$).
- The semantics is given by a set of admissible correctness valuations V^*_{LP}, each member of which is induced by the admissible valuations of LP via C1* and C2.

LP*

- It validates many of the classical rules given in (Smiley 1996) and (Rumfitt 2000), but not all. For instance, the following two classically valid rules for negation

$$+\text{-}\neg\text{-E}\ \frac{\Gamma \vdash +(\neg A)}{\Gamma \vdash -A} \qquad -\text{-}\neg\text{-I}\ \frac{\Gamma \vdash +(A)}{\Gamma \vdash -(\neg A)}$$

- are LP*-invalid.

LP*

$$+\text{-}\neg\text{-E}\ \frac{\Gamma \vdash +(\neg A)}{\Gamma \vdash -A} \qquad -\text{-}\neg\text{-I}\ \frac{\Gamma \vdash +(A)}{\Gamma \vdash -(\neg A)}$$

- The assertibility of $\neg A$ doesn't entail the deniability of A, for if A is both true and false, then $\neg A$ is correctly assertible, but A is not correctly deniable.
- Similarly, if the assertion of A is correct, it doesn't follow that the denial of $\neg A$ is also correct: if A is, once again, both true and false, the assertion of A is correct but its denial isn't.

LP*

On the other hand, the semantics validates the following, highly intuitive, coordination principles:

$$\text{Coord}_1\ \frac{\Gamma, +A \vdash}{\Gamma \vdash -A} \qquad \text{Coord}_2\ \frac{\Gamma \vdash +A \qquad \Delta \vdash -A}{\Gamma, \Delta \vdash}\,.$$

LP*

$$\text{Coord}_1 \ \frac{\Gamma, +A \vdash}{\Gamma \vdash -A}$$

- Coord_1 is a form of *reductio*: it effectively tells us that if A and all the members of Γ cannot be all correctly asserted, then asserting all the members of Γ will warrant the denial of A.
- Priest comes close to endorsing the principle in the following passage:

> *An argument against an opponent who holds A to be true is rationally effective if it can be demonstrated that A entails something that ought rationally to be rejected B. For, it then follows that they ought to reject A. (Priest 2006, p. 86)*

LP*

$$\text{Coord}_2 \; \frac{\Gamma \vdash +A \qquad \Delta \vdash -A}{\Gamma, \Delta \vdash} \; .$$

- Coord_2 tells us that if Γ and Δ warrant the assertion *and* the denial of A, then one may *not* correctly assert all the members of Γ and Δ.
- That is, Coord_2 expresses the highly plausible principle, which leading glut theorists endorse, that one may not correctly assert and deny the same proposition.
- As Priest puts it:

  it is impossible jointly to accept and reject the same thing . . . acceptance and rejection are mutually incompatible. (Priest 2006a, p. 103)

Deniability and LP*

Suppose the language is rich enough to express *deniability*. That English contains some such predicate seems beyond doubt, as the following examples show:

(*) The judge is confident that everything Marc said is deniable.

(**) If what I say is deniable, why is nobody objecting?

LP*

Where T is a dialetheic extension of PA, with underlying logic LP*, let us add to T's language a fresh predicate $\mathcal{D}(x)$ expressing correct deniability; such that:

$\mathcal{D}(\ulcorner A \urcorner)$ is true iff A is correctly deniable.

Then, $\mathcal{D}(x)$ will at least satisfy the following:

(D) $v(\mathcal{D}(\ulcorner A \urcorner)) = 1$ *iff* $v_c(-A) = \boldsymbol{1}$.

LP*

It is easy to check that, given Coord_1 and Coord_2, the following rules governing the deniability predicate must then hold:

$$\mathcal{D}\text{-I}\ \frac{\Gamma, +A \vdash}{\Gamma \vdash +\mathcal{D}(\ulcorner A \urcorner)} \qquad \mathcal{D}\text{-E}\ \frac{\Gamma \vdash +A \qquad \Delta \vdash +\mathcal{D}(\ulcorner A \urcorner)}{\Gamma, \Delta \vdash}\ .$$

LP*

- The rules have intrinsic intuitive appeal.

$$\mathcal{D}\text{-I}\ \frac{\Gamma, +A \vdash}{\Gamma \vdash +\mathcal{D}(\ulcorner A \urcorner)} \qquad \mathcal{D}\text{-E}\ \frac{\Gamma \vdash +A \qquad \Delta \vdash +\mathcal{D}(\ulcorner A \urcorner)}{\Gamma, \Delta \vdash}\ .$$

- The first says that, if A and all the members of Γ cannot be all correctly asserted, then asserting all the members of Γ will warrant the assertion that A is deniable.
- The second tells us that one may not correctly assert both that A and that A is deniable.
- With these rules in place, it is now possible to show that T is incoherent, i.e. that it licenses both the assertion and the denial of the same sentence.

The Paradox of Deniability, in LP*

Let D be a sentence which say of itself (only) that is deniable or rejectable, our (R)

Then, D satisfies:

$$\frac{+D}{+\mathcal{D}(\ulcorner D \urcorner)} \qquad \frac{+\mathcal{D}(\ulcorner D \urcorner)}{+D}$$

The Paradox of Deniability, in LP*

One may then reason thus:

$$
\dfrac{\dfrac{\text{(D), C2}\ \dfrac{\dfrac{+D \vdash +D}{+D \vdash +\mathcal{D}(\ulcorner D \urcorner)}}{+D \vdash -D} \qquad +D \vdash +D}{+D, +D \vdash}\ \text{Coord}_2}{\dfrac{+D \vdash}{\vdash +\mathcal{D}(\ulcorner D \urcorner)}\ \mathcal{D}\text{-I}}\ \text{Contraction}
$$

D is both assertible and deniable.

The Paradox of Deniability, in LP*

Similar paradoxes can be generated for any predicate Θ satisfying Θ-analogues of $\mathcal{D}$-I and $\mathcal{D}$-E, such as 'is rejectable', 'is not rationally acceptable' etc.

A formal version of the *Paradox of Rejectability*.

The Paradox of Rejectability, in LP*

- To see why the foregoing reasoning is a paradox, it is enough to reflect on the fact that the assumptions required to 'prove' $+D$ and $-D$ are quite minimal indeed:
- the standardly accepted structural rules, some means of generating self-reference, the claim that *denial* is exclusive, as codified by C2, Coord_2, and (D), viz. the claim that the *deniability predicate* expresses an exclusive notion of denial.

The Paradox of Rejectability, in LP*

Assuming that self-reference isn't the culprit, glut theorists who accept the standard structural rules are left but with two uncomfortable options:

- either deny that denial is exclusive,
- or deny that exclusive denial is expressible.

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Norms of Denial

As said, we have two options:

- deny that denial is exclusive, i.e. drop Coord_2 *and* substitute C2 with C2*

$$\text{(C2)}\ \ v_c(-A) = \mathbf{1} \text{ iff } v(A) = 0.$$

$$\text{(C2}^*\text{)}\ \ v_c(-A) = \mathbf{1} \text{ iff } v(A) \subseteq \{0, 0.5\}$$

- denying that exclusive denial is expressible.

Norms of Denial

But deniability *is* expressible in the language as the examples given before have shown.

(*) The judge is confident that everything Marc said is deniable.

(**) If what I say is deniable, why is nobody objecting?

So, the only option is to substitute C2 with C2*.

Norms of Denial

- Real issue: can denial in a dialetheist framework be exclusive?
- Let us assume that it is.
- Then, one obvious difficulty arises as soon as one tries to express norms for exclusive denial – such as the one codified by C2 – in the object language.
- The norm asserts that A is correctly deniably iff A is *false only*.

Norms of Denial

The norm asserts that A is correctly deniable iff A is *false only*.

But obviously such a norm, in its intended interpretation, cannot be expressed in the dialetheist's language:

falsity only just is Boolean negation – a notion that, on the foregoing assumptions *must* be deemed incoherent.

So, perhaps, denial isn't exclusive after all.

Norms of Denial

Indeed, Priest sometimes questions the exclusivity of denial. He considers two norms of denial, neither of which makes denial exclusive. The first one is the following:

> **Deny(U)** *You may deny A if there is good evidence for A's untruth,*

A plausible corresponding norm for assertion:

> **Assert(T)** *You may assert A if there is good evidence for A's truth,*

but the existence of sentences, such as the Liar sentence, that can be proved to be both true and untrue, **Deny(U)** licenses us to accept and reject the same sentence.

Norms of Denial

But, if **Deny(U)** licenses us to accept and reject the same sentence Lisa's denial of the sentence Padua is North of Venice would no longer be guaranteed to express disagreement, since the assertion and denial of:

- Padua is North of Venice.

could then *both* be correct.

More specifically, if the assertion and the denial of A can both be correct, just like A and $\neg A$ can both be true, the denial of A cannot express disagreement with the assertion that A.

Norms of Denial

Lisa's assertion is *not* guaranteed to express disagreement with Marc's assertion.

If denial is to serve as a means to express disagreement, it must be *rationally impermissible* to both assert and deny A.

That is, denial must be exclusive: there may not be overlap between assertibility and deniability.

Norms of Denial

But observe that:

In view of the Paradox of Deniability, however, no comprehensive set of norms for exclusive denial can be formulated in the glut-theorist's language.

Norms of Denial

- Plausibly, such norms would have to register the fact that correct denial ‘aims’ at falsity only, just like assertion aims at truth.
- Hence, they must involve notions that cannot be expressed, on pain of incoherence.
- This makes denial in LP* somewhat ineffable: in absence of an appropriate norm, it is hard to see, for instance, how incorrect denials can be criticised.
- One can deem denials as correct, or incorrect. But one cannot say *why* a given denial was correct.

Norms of Denial

Priest also considers a second norm:

Deny(U)* *You may deny A if there is good evidence for A's untruth, unless there is also good evidence for its truth.*

In short: we may deny A if we have good reasons for thinking that A is untrue *only*.

Norms of Denial

- This second norm, though, makes denial profoundly *unlike* assertion.
- Problem: Unlike assertion, any denial may later turn out to be incorrect, since any false sentence can in principle be discovered to be a glut.

Norms of Denial

- Thus, you can disagree with my assertion that $0 \neq 0$, and thus deny $0 \neq 0$. But, even if you can prove $0 = 0$, and hence disprove $0 \neq 0$, you can never be fully confident that your denial is correct: a proof of $0 \neq 0$ may always turn up.
- By contrast, if you have proved $0 = 0$ and thereby assert it, you can be fully confident that your assertion is correct.

Norms of Denial

That is a problematic asimmetry:

nothing in our practice of asserting and denying things suggests that assertion can be indefeasible in a way that denial is not.

Norms of Denial. A conclusion

- Exclusive semantic notions are needed to express norms governing exclusive denial.
- In turn, exclusive denial would appear to be needed, in a glut-theoretic framework, in order to express disagreement.
- But if no norms for exclusive denial can be non-trivially formulated in such a framework, the claim that exclusive denial can serve as a means to express disagreement would appear to loose much of its initial appeal.

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  > *attempts to formulate distinctive Liar paradoxes in the form of denial fail, since [−] being a force operator, has no interaction with the content of what is uttered (Priest 2006, p. 108).*

- We (of course) agree!
- The deniability predicate is, precisely, not a force operator.

♯2 Dropping $\mathcal{D}$-E?

$$\mathcal{D}\text{-I}\ \frac{\Gamma, +A \vdash}{\Gamma \vdash +\mathcal{D}(\ulcorner A \urcorner)} \qquad \mathcal{D}\text{-E}\ \frac{\Gamma \vdash +A \qquad \Delta \vdash +\mathcal{D}(\ulcorner A \urcorner)}{\Gamma, \Delta \vdash}\ .$$

- It may be objected that $\mathcal{D}$-E must be invalid.

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- It may be objected that $\mathcal{D}$-E must be invalid.
- There's surely valuations in V that make both A and $\mathcal{D}(\ulcorner A \urcorner)$ true.
- This valuation, however, is ruled out by the principle—which dialetheists accept (Priest 2010)—that we may not deny what's true.

♯2 Dropping $\mathcal{D}$-E?

That no such evaluation is possible is a consequence of C2 togheter with (D).

(C2) $v_c(-A) = \mathbf{1}$ iff $v(A) = 0$.

(D) $v(\mathcal{D}(\ulcorner A \urcorner)) = 1$ iff $v_c(-A) = \mathbf{1}$.

♯2 Dropping $\mathcal{D}$-E?

Rejecting either principle would seem problematic:

(D) $v(\mathcal{D}(\ulcorner A \urcorner)) = 1$ iff $v_c(-A) = \mathbf{1}$.

(D) ensures that the deniability predicate expresses *exclusive* denial: it guarantees that A is deniable is correct if and only if A is, in effect, deniable.

♯2 Dropping $\mathcal{D}$-E?

(C2) $v_c(-A) = \mathbf{1}$ iff $v(A) = 0$.

C2, on the other hand, codifies the exclusivity of denial: it guarantees that, if A is, in effect deniable, then A is false only (and vice versa).

♯2 Dropping $\mathcal{D}$-E?

Priest effectively concedes that in ‘normal conditions’ the denial of A may be expressed by asserting $A \to \perp$, where $\to$ is a detachable conditional (Priest 2006, p. 105) and $\perp$ is a ‘a logical constant such that it is a logical truth that $\perp \to A$, for every A’ (Priest2006, p. 85).

$\sharp$2 Dropping $\mathcal{D}$-E?

What Priest intends by "normal conditions" and "most contexts" is explained in the following quotation:

> *In most contexts, an assertion of [...] $\alpha \rightarrow \perp$ would constitute an act of denial. Assuming that the person is normal, they will reject $\perp$, and so, by implication, α. The qualifier "in most contexts" is there because if one were ever to come across a trivialist who accepts $\perp$, this would not be the case. For such a person an assertion of [$\alpha \rightarrow \perp$] would not constitute a denial: nothing would. (Priest 2006, p. 105-106)*

♯2 Dropping $\mathcal{D}$-E?

Simply put, Priest's idea is this:

- if $\perp$ entails triviality, then *in most contexts* an assertion of $A \rightarrow \perp$ will be equivalent to the exclusive denial of A.
- For, in most contexts, there will be no trivialists, and, since $\rightarrow$ detaches, an assertion of A in presence of $A \rightarrow \perp$ would entail what in most contexts is never accepted, viz., triviality.
- But, in some contexts there will be trivialists – speakers who accept everything, including $\perp$. Hence, the assertion of $A \rightarrow \perp$ cannot be equivalent to the exclusive denial in general.

♯2 Dropping $\mathcal{D}$-E?

Thus, if we interpret $\mathcal{D}(\ulcorner A \urcorner)$ as $A \to \bot$, in most contexts (D) and (C2) hold:

(D) $v(\mathcal{D}(\ulcorner A \urcorner)) = 1$ iff $v_c(-A) = \mathbf{1}$.

(C2) $v_c(-A) = \mathbf{1}$ iff $v(A) = 0$.

$\mathcal{D}(\ulcorner A \urcorner)$ is true only if A is false only, as the Paradox of Deniability assumes.

Glut theorists who reject trivialism and accept *modus ponens* for $\to$ must reject A, it they accept $A \to \bot$.

Thus, if the glut theorists understands $\mathcal{D}(\ulcorner A \urcorner)$ as equivalent to $A \to \bot$, then $\mathcal{D}$-E, and hence the Paradox of Deniability, would be validated.

♯3 Dropping $\mathcal{D}$-I?

$$\mathcal{D}\text{-I}\ \frac{\Gamma, +A \vdash}{\Gamma \vdash +\mathcal{D}(\ulcorner A \urcorner)} \qquad \mathcal{D}\text{-E}\ \frac{\Gamma \vdash +A \qquad \Delta \vdash +\mathcal{D}(\ulcorner A \urcorner)}{\Gamma, \Delta \vdash}\ .$$

- It may be objected that $\mathcal{D}$-I isn't valid in LP*.
- However, if knowing that A entails triviality isn't a good enough ground for asserting that A is deniable, one wonders whether there can ever be such grounds.

♯4. A More general objection. On rules

One could finally argue that, following Priest, a norm for denial:

> *may be understood as an acceptable default rule, but not as an indefeasible one. (2006b, 110)*

♯4. A More general objection. On rules

- Observe that if this explanation were taken seriously, one would never be justified to deny / reject any sentence.
- For, one can never rule out a priori the existence of grounds for its truth.

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Conclusions

Denial plays a key role in the standard dialetheist account of disagreement: glut theorists may express disagreement with someone's assertion that A by *denying* A, or so the account goes.

In order for this to work, denial must be exclusive: one may not correctly assert and deny the same proposition.

Conclusions

LP can be expanded into a bilateral logic of exclusive denial, LP^*.

However, as shown by the Paradox of Deniability, *exclusive deniability* – a key semantic notion of the logic – is not expressible in the dialetheist's language.

Conclusions

Glut theorists are faced with a dilemma:

- either denial can serve as means to express disagreement, but the notion of exclusive deniability isn't expressible in the glut theorist's language,
- or deniability is expressible, but denial may no longer serve as a means to express disagreement.

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