Beyond “knowing that”:
V. Conclusions, discussions and future directions

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ESSLLI 2015
Summary

Discussions and future work
Recap: agenda

- Take a know-X construction as a single modality.
- Give an intuitive semantics according to *some* interpretation. There may be many!
- Axiomatize the logics with (combinations of) those operators.
- Dynamify those logic with knowledge updates.
- Automate the inferences based on decidability.
- Come back to philosophy and linguistics with new insights.
What we have covered

- Knowing whether (non-contingency operator)
  - Succinctness and convenience of using “knowing whether”
  - Non-contingency in different settings
  - almost-definability schema AD
  - Δ-bisimulation
  - Undefinablility of frame properties
  - Characterization theorems
  - Basic system with three special axioms
  - Extended systems over various classes of frames
  - The technique of using AD to build canonical model and to find appropriate axioms
  - The technique of building two maximal consistent sets for truth lemma
  - Reduction axioms for dynamic knowing whether

- “Ignorance is contingent.” Do talk to people a lot to find related work: google may not tell you everything.
What we have covered

- Knowing what
  - Sum & Product and $K_v$ operator
  - Interaction of knowing that and knowing what
  - First-order epistemic model with constant domain
  - Incompleteness and ETL-like semantics for DEL-like logic
  - PALK$^v$ cannot be reduced to ELK$^v$ via d-bisimulation
  - Conditional $K_v$ operator
  - The axiomatization with a crucial axiom handling the composition of conditions
  - Canonical model with saturated MCSs (counterparts of some FO-model formulas)
  - Controlling the extra information
  - Simultaneously building two consistent sets
  - ELK$^{vr}$ is equally expressive as PALK$^v$ and PALK$^{vr}$
  - Decidability by bounded number of needed values

- Do read classic papers and try to find something interesting.
What we have covered

- Knowing how
  - In-between knowledge-that and ability
  - Tricky examples in philosophy literature
  - We focus on goal-direct knowing how
  - Language: condition and guarantee
  - Semantics: conformant planning
  - Model: labelled transition system
  - Non-local feature for knowing how
  - Definable universal modality $U$ for background knowledge
  - Axiomatization using $U$ and the axiom of compositionality
  - Different canonical model for different consistent sets
  - Sets of labels in Canonical model using condition and goal pairs
  - One-step plans in Canonical model
  - Inductive proof for "existence lemma"

- Ideas from different fields may bring new insights.
Beyond knowing that: difficulties

- not normal: not really a big deal and Kripke models may just be enough and intuitive.
- weak language vs. rich model: add information in canonical models if necessary.
- new axioms: find them with syntactic and semantic insights, and fill the gaps in completeness proofs.

Each Know-X has its own features!
Other knowing-X

- knowing why the shadow of the pole is 10m long; knowing why he comes to ESSLLI... (ongoing with Chao Xu):
  - Motives
  - Scientific explanation: causal, deductive, statistical, mechanistic...
- knowing who the first speaker is; knowing who skipped the course (ongoing):
  - he is Mike my colleague
  - he is the president of the university (but I don’t know who the president of university is)
- knowing which courses you liked the best
- knowing when you graduated
- knowing where you were born
- ...

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Recap: some warnings

- idealized agents
- sometimes single-agent
- no group notions
- not so much about dynamics
- no true agency
- not fully compositional in the sense of linguistics
- mainly ‘knowledge’ rather than ‘belief’, no probability

See these as opportunities.
Group notions

- Knowing that: general knowledge, common knowledge, distributed knowledge. Let $O_G \phi$ as $\bigwedge_{i \in G} O_i \phi$.
- Common knowledge regarding knowing whether $\phi$: $C_G Kw_G \phi$?
  $C_G \phi \lor C_G \neg \phi$? $Kw_G Kw_G \ldots \phi$? $C_G \bigwedge_{i \in G} (Kw_G K_i \phi)$?
  $C_G \bigwedge_{i \in G} (Kw_G Kw_i \phi)$? (whether know vs. know whether) ...
- Common knowledge of what $c$ is: $C_G Kv_G c$? $Kv^*_G c$?
- Common knowledge of how to achieve $\phi$: $C_G Kh_G \phi$ (do we require the same plan?)
- Collaborative know-how, competitive know-how.
Nested knowledge

- Complete ignorance (via Hans van Ditmarch):
  \( \neg Kw_i p \land \neg Kw_i Kw_j p \land \neg Kw_i Kw_j Kw_i p \ldots ? \)

- I know what \( c \) is given that I know what \( d \) is \( Kv(Kvd, c) \)?
  (Ongoing work with Ding)

- Do these make sense? \( Kh_i Kh_i p \) ? \( Kh_i Kh_j \phi \) ? \( Kwhy_i Kwhy_i \phi \) ? \( Kwhy_i Kwhy_j \phi \) ?
(Logical) omniscience

- inherited omniscience due to knowing that in knowing whether
- unlimited planning power in knowing how
- Frege’s puzzle: semantically $a = b \land Kva \land Kvb \rightarrow K(a = b)$
Dynamics and interaction of different type of knowledge

- Announcing whether / what / how / why \( \phi \) (Malvin); links to inquisitive dynamic epistemic logic.
- Knowing the password helps to know how to get the money.
- Knowing how to open the box helps to know what it is inside.
- I know how you know it. I know why you believe it.
Some technical questions

- Correspondence theory with FOL logic in frame definability;
- Direct axiomatization of \( \text{PALK}^v \);
- Model theory of those logics;
- Using axioms as definitions of new modalities;
- Guarded fragments of FO-modal logic.

There are just too many things to do... More suggestions?
Summary

Goals of this course

- Attention to knowing-X expressions
- Be aware of the relevant conceptual questions
- Be aware of some relevant techniques
- Ideas and thinking process behind
- Learn from you!

I believe this is just the beginning of an interesting story.
Beyond “knowing that”: join us!

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