Conditionally knowing what

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Epistemic Logic

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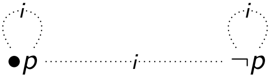
$$p \wedge \neg K_i p$$



$\mathbb{S}5$ system

System $\mathbb{S}5$

Axioms		Rules	
TAUT	all the instances of tautologies	MP	$\dfrac{\phi, \phi \to \psi}{\psi}$
DISTK	$K_i(p \to q) \to (K_i p \to K_i q)$	NECK	$\dfrac{\phi}{K_i \phi}$
T	$K_i p \to p$	SUB	$\dfrac{\phi}{\phi[p/\psi]}$
4	$K_i p \to K_i K_i p$		
5	$\neg K_i p \to K_i \neg K_i p$		

Theorem

$\mathbb{S}5$ *is sound and strongly complete for modal logic over S5 frames.*

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Logically: how to reason about those forms of knowledge?

Computationally: how to efficiently represent and do inference about them?

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- Dynamify those logic with knowledge updates.
- Automate the inferences.
- Come back to philosophy and linguistics with new insights.

Beyond knowing that: difficulties and some results

New operator behaves quite differently from the standard modal operators and are usually disguised FO-modal fellows, which causes difficulties for axiomatization and decidable machinery.

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- Knowing how: alternative non-possible-world semantics [Wang LOFT12]; extend [Wang & Li AiML 12] for epistemic planning

Knowing what operator Kv_i proposed by [Pla89]

ELKv is defined as (where $c \in C$):

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$$\boxed{\begin{array}{lcl} \mathcal{M}, s \vDash Kv_i c & \iff & \text{for any } t_1, t_2: \text{ if } s \sim_i t_1, s \sim_i t_2, \\ & & \text{then } V_C(c, t_1) = V_C(c, t_2). \end{array}}$$

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The interaction between the two operators is crucial: it cannot be treated as $K_i K_j p \wedge \neg K_i p$.

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To handle the Sum and Product puzzle, Plaza extended **ELKv** with announcement operator (call it **PALKv**):

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Theorem ([WF13])

$\langle p\rangle Kv_i c \wedge \langle q\rangle Kv_i c \to \langle p \vee q\rangle Kv_i c$ is not derivable in $\mathbb{PALKV}_p$, thus $\mathbb{PALKV}_p$ is not complete w.r.t. $\vDash$ on epistemic models.

Conditionally knowing what

Axiomatizing **PALKv** is indeed hard. We propose a conditional generalization of Kv_i operator:

$$\phi ::= \top \mid p \mid \neg\phi \mid \phi \wedge \phi \mid K_i\phi \mid Kv_i(\phi, c)$$

where $Kv_i(\phi, c)$ says 'agent i knows what c is *given* ϕ', e.g., I know my password for this website if it is 4-digit. More precisely, agent i *would know* what c is if he is informed that ϕ.

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$$\mathcal{M}, s \vDash Kv_i(\phi, c) \quad \Leftrightarrow \quad \text{for any } t_1, t_2 \in S \text{ such that } s \sim_i t_1 \text{ and } s \sim_i t_2 : \mathcal{M}, t_1 \vDash \phi \,\&\, \mathcal{M}, t_2 \vDash \phi \text{ implies } V_C(c, t_1) = V_C(c, t_2)$$

Let **PALKv**^{r} be:

$$\phi ::= \top \mid p \mid \neg\phi \mid \phi \wedge \phi \mid K_i\phi \mid Kv_i(\phi, c) \mid \langle\phi\rangle\phi$$

PALKv^r looks more expressive than **PALKv** but in fact they are equally expressive.

Theorem ([WF13])

The comparison of the expressive power of those logics are summarized in the following (transitive) diagram:

$$\begin{array}{ccc} \mathbf{ELKv}^r & \longleftrightarrow & \mathbf{PALKv}^r \\ \uparrow & & \updownarrow \\ \mathbf{ELKv} & \longrightarrow & \mathbf{PALKv} \end{array}$$

where **ELKv** and **ELKv**^{r} are the announcement-free fragments of **PALKv** and **PALKv**^{r}.

We can simply forget about Plaza's **PALKv** and use **ELKv**^{r}!

System $\mathbb{ELKV}^r$

Axiom Schemas

TAUT	all the instances of tautologies
DISTK	$K_i(p \to q) \to (K_i p \to K_i q)$
T	$K_i p \to p$
4	$K_i p \to K_i K_i p$
5	$\neg K_i p \to K_i \neg K_i p$
DISTKv^r	$K_i(p \to q) \to (Kv_i(q, c) \to Kv_i(p, c))$
Kv^{r}4	$Kv_i(p, c) \to K_i Kv_i(p, c)$
Kv$^r\bot$	$Kv_i(\bot, c)$
Kv$^r\vee$	$\hat{K}_i(p \wedge q) \wedge Kv_i(p, c) \wedge Kv_i(q, c) \to Kv_i(p \vee q, c)$

Rules

MP	$\dfrac{p, p \to q}{q}$
NECK	$\dfrac{p}{K_i p}$
SUB	$\dfrac{\phi}{\phi[p/\psi]}$
RE	$\dfrac{\psi \leftrightarrow \chi}{\phi \leftrightarrow \phi[\psi/\chi]}$

$Kv_i(\phi, c)$ can be viewed as $\exists x K_i(\phi \to c = x)$ where x is a *rigid* variable and c is a *non-rigid* one.

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- counterparts of atomic formulas such as $c = x$
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By using axioms in the modal language, we need to make sure these extra bits are consistent with the maximal consistent sets and canonical relations.

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Each saturated MCS including $\neg Kv_i(\phi, c)$ has two saturated ϕ-successors which disagree about the value of c.

Axiom $\mathtt{Kv}^r\vee$: $\hat{K}_i(p \wedge q) \wedge Kv_i(p, c) \wedge Kv_i(q, c) \to Kv_i(p \vee q, c)$ plays an extremely important role.

Theorem

$\mathbb{ELKV}^r$ *is sound and strongly complete for* $\mathbf{ELKv}^r$.

Theorem ([Xio14])

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Theorem

$\mathbb{ELKV}^r$ *is sound and strongly complete for* **ELKv**^{r}.

Theorem ([Xio14])

ELKv^r *on epistemic models is decidable.*

We can axiomatize multi-agent **PALKv**^{r} by adding the following reduction axiom schemas (call the resulting system $\mathbb{SPALKV}^r$):

!ATOM	$\langle\psi\rangle p \leftrightarrow (\psi \wedge p)$
!NEG	$\langle\psi\rangle\neg\phi \leftrightarrow (\psi \wedge \neg\langle\psi\rangle\phi)$
!CON	$\langle\psi\rangle(\phi \wedge \chi) \leftrightarrow (\langle\psi\rangle\phi \wedge \langle\psi\rangle\chi)$
!K	$\langle\psi\rangle K_i\phi \leftrightarrow (\psi \wedge K_i(\psi \rightarrow \langle\psi\rangle\phi))$
!Kv^r	$\langle\phi\rangle Kv_i(\psi, c) \leftrightarrow (\phi \wedge Kv_i(\langle\phi\rangle\psi, c))$

Corollary

$\mathbb{SPALKV}^r$ *is sound and strongly complete for* **PALKv**^{r}.

Final words

Systematic study of "knowing-wh" constructions in logic may lead us to:

- interesting non-normal 'modal' operators and axioms
- discovery of new decidable ("guarded") fragments of first-order modal logic
- knowledge representations closer to natural language
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This is just the *beginning* of an interesting story!

Beyond "knowing that": Join us!

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