Epistemic Informativeness

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Frege's puzzle on identity statements (Frege (1892))

How do we explain the difference between $a = a$ and $a = b$ in cognitive value when a and b are co-referential?

E.g., let $a = \textit{Phosphorus}$, $b = \textit{Hesperus}$. They both refer to the planet Venus, but $a = a$ and $a = b$ seem to express different things.

Frege thought his had solved the puzzle by arguing co-referring (proper) names may have different *senses*. Ruth Barcan Marcus, Saul Kripke and others denied that a proper name has sense and developed Millian theory of direct reference further.

What is the concept of ‘cognitive value’(CV)?

From the literature:

- (Salmon, 1986, p.13): CV is the piece of information the sentence encodes.
- (Taschek, 1992, p.773): Differing in CV if it is possible to believe one and disbelieve (or has no judgement on) the other.
- (Yagisawa, 1993, p.136): A sentence is informative iff it is not analytic.
- (Fine, 2007, p.34): Difference in CV means conveying different information to a person who understands the sentences.

Many attribute the *differences* in cognitive value to the differences in *informativeness*. Again, what is this ‘informativeness’?

What is the concept of ‘cognitive value’?

It is hard to define the very vague concepts of ‘cognitive value’ and ‘informativeness’, but we can try to define a more restricted concept which can be made precise.

Frege (1892): ‘$a = a$ holds *a priori* … while statements of the form $a = b$ often contain very valuable *extensions* of our *knowledge* …’.

1. Whose knowledge are we talking about?
2. How does the statement extend our knowledge?

Some observations:

1. The agent’s current (knowledge) state matters.
2. Just adding the new statement is not enough.

Epistemic Informativeness: the intuitive idea

The epistemic informativeness (EI) of a true statement expressing ϕ to an agent given a concrete situation is the set of new propositions that the agent comes to know after he is informed that ϕ.

- We are defining a particular type of informativeness of an *arbitrary* true statement.
- We are not defining CV, but a more restricted notion: *differences* in EI should imply *differences* in CV, but maybe not the other way around.

Similar ideas: Fiengo and May (2002), Rendsvig (2012)

Is EI an useful notion?

Can this notion of EI help to explain the following strengthened version of Frege's puzzle?

> *How to explain the difference between $a = a$ and $a = b$ in cognitive value even when they are both true and the agent* **already knows** *that $a = b$.*

In a multi-agent setting, it may still work: although I know ϕ already, but others might not know, and if ϕ is communicated to all then it may extend my knowledge about others' knowledge in ϕ with extra conditions.

- What about the case when all the agents know ϕ?
- When exactly do two statements have the same EI (given the same condition)?
- Now we need to be precise.

How to formalize EI precisely?

We need to talk about the knowledge of agents and how the knowledge is updated when receiving true statements.

Public Announcement Logic (**PAL**) proposed by Plaza (1989).

$$\phi ::= p \mid \neg\phi \mid \phi \wedge \phi \mid K_i\phi \mid C\phi \mid \langle\phi\rangle\phi$$

- $K_i\phi$ expresses that 'agent i knows that ϕ'
- $C\phi$ reads 'ϕ is common knowledge among all the agents'.
- $\langle\phi\rangle\psi$ says after the truthful announcement of ϕ, ψ holds.

Semantics

The meaning of an announcement is the change it brings to the epistemic states of all the agents.

The language of **PAL** is interpreted on pointed S5 Kripke models $\mathcal{M}_w = \langle W, \{\sim_i \mid i \in \mathbf{I}\}, V, w\rangle$:

$$
\begin{array}{rcl}
\mathcal{M}_w \vDash p & \Leftrightarrow & p \in V(w) \\
\mathcal{M}_w \vDash \neg\phi & \Leftrightarrow & \mathcal{M}_w \nvDash \phi \\
\mathcal{M}_w \vDash \phi \wedge \psi & \Leftrightarrow & \mathcal{M}_w \vDash \phi \text{ and } \mathcal{M}_w \vDash \psi \\
\mathcal{M}_w \vDash K_i\psi & \Leftrightarrow & \text{for all } v \text{ such that } w \sim_i v : \mathcal{M}_v \vDash \psi \\
\mathcal{M}_w \vDash C\psi & \Leftrightarrow & \text{for all } v \text{ such that } w \sim^* v : \mathcal{M}_v \vDash \psi \\
\mathcal{M}_w \vDash \langle\psi\rangle\phi & \Leftrightarrow & \mathcal{M}_w \vDash \psi \text{ and } (\mathcal{M}|_\psi)_w \vDash \phi
\end{array}
$$

where $\sim^*$ is the transitive closure of $\bigcup_{i \in \mathbf{I}} \sim_i$ and $\mathcal{M}|_\psi = (W', \{\sim'_i \mid i \in \mathbf{I}\}, V')$ where:

$$W' = \langle v \mid \mathcal{M}_v \vDash \psi\rangle, \sim'_i = \sim_i |_{W' \times W'}, V' = V|_{W'}$$

Epistemic Informativeness: a formal definition

Definition

Given a pointed model $\mathcal{M}_w$ which satisfies ϕ, the epistemic informativeness (EI) of ϕ to agent i at $\mathcal{M}_w$ is the set (denoted as $ei(\phi, i, \mathcal{M}_w)$):

$$\{\chi \mid \mathcal{M}_w \vDash \neg K_i\chi \wedge \langle\phi\rangle K_i\chi\}$$

Based on this, we can define the *conditional* EI of ϕ to agent i given the assumption set Δ of **PAL** formulas (denoted as $ei(\phi, i, \Delta)$):

$$\{\langle\mathcal{M}_w, \Gamma\rangle \mid \mathcal{M}_w \vDash \Delta \cup \{\phi\} \text{ and } \Gamma = ei(\phi, i, \mathcal{M}_w)\}$$

Theorem

For any set Δ of **PAL** *formulas and any basic facts ϕ, ψ:*

$$ei(\phi, i, \Delta) = ei(\psi, i, \Delta) \iff \Delta \vDash [\phi \vee \psi]C(\phi \leftrightarrow \psi).$$

Remark

- $[\phi \vee \psi]$ *is needed:*

$$\mathbf{w} : p, q \overset{1}{\longleftrightarrow} \neg p \neg q \overset{2}{\longleftrightarrow} p, \neg q$$

  $ei(p, 1, \mathcal{M}_w) = ei(q, 1, \mathcal{M}_w)$, *but* $\mathcal{M}_w \nvDash C(p \leftrightarrow q)$.
- *Agent i disappeared at the right hand side of the equivalence (due to the validity of $C\chi \leftrightarrow K_i C\chi$).*
- $[\phi \vee \psi]C(\phi \leftrightarrow \psi)$ *can be viewed as relativized common knowledge introduced in van Benthem et al. (2006).*

Theorem

For any set Δ of **PAL** *formulas and any Boolean ϕ, ψ:*

$$ei(\phi, i, \Delta) = ei(\psi, i, \Delta) \iff \Delta \vDash [\phi \vee \psi] C(\phi \leftrightarrow \psi).$$

Let $\Delta_n^\psi = \{K_{i_1} K_{i_2} \cdots K_{i_k} \psi \mid k \leq n, i_j \in \mathbf{I}\}$, in particular $\Delta_0^\psi = \{\psi\}$ then the following corollary is immediate, based on the observation that if ϕ is valid then $\phi \vee \psi$ is valid and $\phi \leftrightarrow \psi$ is equivalent to ψ.

Corollary

For any Boolean formulas ϕ and ψ, any set of **PAL** *formulas Δ, if ϕ is valid then:*
(1) $ei(\phi, i, \Delta) = ei(\psi, i, \Delta) \iff \Delta \vDash C\psi$
(2) $ei(\phi, i, \Delta_n^\psi) \neq ei(\psi, i, \Delta_n^\psi)$ for all $n \in \mathbb{N}$ if $\nvDash C\psi$.

To apply the corollary to equalities, Two 'innocent' assumptions are needed:

- A $a = b$ and $a = a$ express *some* propositions about basic facts, no matter what exactly they are.
- B the proposition expressed by $a = a$ is valid.

Let ϕ be the proposition expressed by $a = a$ and ψ be the proposition expressed by $a = b$ then from the corollary:

(1) $ei(\phi, i, \Delta) = ei(\psi, i, \Delta) \iff \Delta \vDash C\psi$
(2) $ei(\phi, i, \Delta^{\psi}_{n}) \neq ei(\psi, i, \Delta^{\psi}_{n})$ for all $n \in \mathbb{N}$ if $\nvDash C\psi$

Conclusion

- We formalize the notion of epistemic informativeness of arbitrary propositional statements and its conditional variant in Public Announcement Logic.
- We show that two statements of basic facts are equally epistemically informative iff the logical equivalence of the two is (weakly) commonly known.
- Differences in EI may help to explain differences in CV.
- As a consequence of the above result, (under two intuitive assumptions) if the proposition expressed by $a = b$ is not commonly known (no matter how close to common knowledge) then $a = b$ and $a = a$ are different in their epistemic informativeness.

We have not solved the puzzle so far

- *What* are the propositions expressed by $a = a$ and $a = b$?
- How to express the *linguistic competence* of an agent?

Based on Wang and Fan (2013):

$$\phi ::= \top \mid a = b \mid \neg\phi \mid \phi \wedge \phi \mid K_i\phi \mid C\phi \mid Kv_i a \mid \langle\phi\rangle\phi$$

- $Kv_i a$ expresses that i knows *what* a is.
- $a = b$ iff the references of a and b are *equivalent*.
- It is possible that $ei(a = a, i, \Delta) \neq ei(a = b, i, \Delta)$ where $\Delta = \{K_i a = b, Kv_i a, Kv_i b\}$.

Thank you very much!

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