

A logic of “knowing how”

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Beyond “knowing that”: a research program

A logic of (one type of) “knowing how”

Conclusions and future work

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Computationally: efficient knowledge representation, and automated reasoning about “knowing X”

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- ▶ Come back to philosophy and linguistics with new insights.

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Some of our results:

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See my ESSLLI course slides on my homepage for more information.

A particular type of “knowing how”

Clarifications:

- ▶ We focus on *goal-direct* “knowing how”: knows how to realize your goal, e.g., I know how to go to Beijing; I know how to make a cake; I know how to get out of the maze.

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- ▶ We do not focus on the philosophical debate between intellectualism (e.g., Stanley & Williamson 2001) and anti-intellectualism (e.g., Ryle 49). See the collection of 200+ papers on the topic at philpaper (edited by Bengson).

Knowing how: starting from scratch

In AI, “knowing how” to achieve a goal is often treated as being able to (or can) reach a goal (Situation Calculus, ATL, STIT). See two excellent surveys: [Gochet 13] and [Ågotnes, Goranko, Jamroga, Wooldridge 15]. Epistemic ATL cannot express it so far.

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- ▶ Even when you can win a lottery by luckily buying the right ticket, it does not mean you know how to win the lottery, since you cannot *guarantee* the result.

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Language and semantics

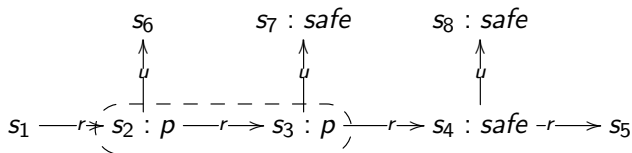
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Example (I know how to be safe given p)

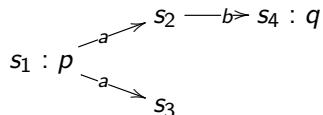


Models

- ▶ Use labelled transition systems to represent your knowledge about your own abilities [Wang ICLA15].

Example

Do you know how to guarantee q given p ?



$$s_1 : p, r \xrightarrow{-a} s_3 \xrightarrow{b} s_5 : q$$

$$s_2 : p \xrightarrow{-b} s_4 \xrightarrow{-a} s_6 : q$$

Putting all these together

The language \mathbf{L}_{Kh} is defined as follows:

$$\varphi ::= \top \mid p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid Kh(\varphi, \varphi)$$

We define $\mathcal{U}\varphi$ as $Kh(\neg\varphi, \perp)$.

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Given a non-empty set of propositional letters \mathbf{P} , a non-empty set of action Σ . A model (ability map) is simply a tuple $(\mathcal{S}, \mathcal{R}, \mathcal{V})$ where:

- ▶ \mathcal{S} is a non-empty set of states;
- ▶ $\mathcal{R} : \Sigma \rightarrow 2^{\mathcal{S} \times \mathcal{S}}$ is a collection of transitions labelled by Σ ;
- ▶ $\mathcal{V} : \mathcal{S} \rightarrow 2^{\mathbf{P}}$ is a valuation function.

Semantics of \mathbf{L}_{Kh}

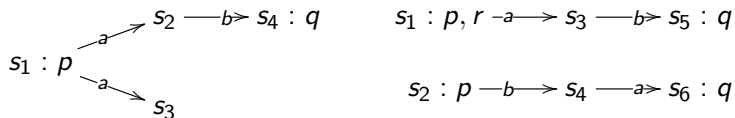
$\mathcal{M}, s \models Kh(\psi, \varphi) \Leftrightarrow$ there *exists* a $\sigma \in \Sigma^*$ such that *for all* $\mathcal{M}, s' \models \psi$:
 (1) σ is *strongly executable* at s' , and
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Quantifier schema: $\exists \forall (\forall \wedge \forall)$ (second order, non-local)

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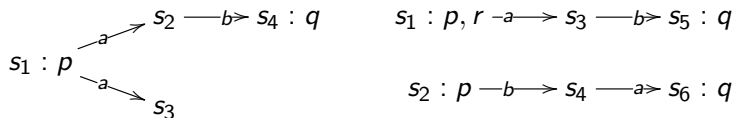


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System SKH

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Axioms

TAUT all axioms of propositional logic

DISTU $\mathcal{U}p \wedge \mathcal{U}(p \rightarrow q) \rightarrow \mathcal{U}q$ COMPKh $\mathcal{K}h(p, r) \wedge \mathcal{K}h(r, q) \rightarrow \mathcal{K}h(p, q)$ EMP $\mathcal{U}(p \rightarrow q) \rightarrow \mathcal{K}h(p, q)$ TU $\mathcal{U}p \rightarrow p$ 4KU $\mathcal{K}h(p, q) \rightarrow \mathcal{U}\mathcal{K}h(p, q)$ 5KU $\neg\mathcal{K}h(p, q) \rightarrow \mathcal{U}\neg\mathcal{K}h(p, q)$ **Rules**MP $\frac{\varphi, \varphi \rightarrow \psi}{\psi}$ NECU $\frac{\psi}{\mathcal{U}\psi}$ SUB $\frac{\varphi(p)}{\varphi[\psi/p]}$

Derivable formulas

TRI	$\mathcal{K}h(p, p)$
WSKh	$\mathcal{U}(p \rightarrow r) \wedge \mathcal{U}(o \rightarrow q) \wedge \mathcal{K}h(r, o) \rightarrow \mathcal{K}h(p, q)$
4U	$\mathcal{U}p \rightarrow \mathcal{U}\mathcal{U}p$
5U	$\neg\mathcal{U}p \rightarrow \mathcal{U}\neg\mathcal{U}p$
COND	$\mathcal{K}h(\perp, p)$
UCONJ	$\mathcal{U}(\varphi \wedge \psi) \leftrightarrow (\mathcal{U}\varphi \wedge \mathcal{U}\psi).$
PREKh	$\mathcal{K}h(\mathcal{K}h(p, q) \wedge p, q).$
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Ongoing work:

- ▶ $\mathcal{K}h(\psi, \varphi) \wedge \neg \mathcal{U}(\psi \rightarrow \varphi)$ which may rule out the weird case of “I know how to digest when eating”
- ▶ Model theory of $\mathbf{L}_{\mathcal{K}h}$
- ▶ Exact complexity
- ▶ Logical omniscience
- ▶ Dealing with how to *maintain* a goal
- ▶ Dealing with “local” know-how
- ▶ Dealing with conditional plans/strategy-based know-how
- ▶ Update of “knowing how”

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