Philosophy of mathematics from working mathematics

Beijing lectures on how the on-going history of mathematics produces ontology and foundations

Colin McLarty, draft in progress December 12, 2017

On the 30th anniversary of Howard Stein's essay "Logos, Logic, and Logistiké: Some Philosophical Remarks on the Nineteenth-Century Transformation of Mathematics."

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Introduction

> Mathematics underwent, in the nineteenth century, a transformation so profound that it is not too much to call it a second birth of the subject—its first birth having occurred among the ancient Greeks, say from the sixth through the fourth century B.C. (Stein, 1988, p. 238)

The theme of these lectures came from Howard Stein's classic essay "Logos, *Logic, and* Logistiké: *Some Philosophical Remarks on the Nineteenth-Century Transformation of Mathematics*" (1988). Stein regarded axiomatic, theoretical mathematics as a unique "capacity of the human mind, or of human thought" which is therefore of "tremendous importance for philosophy" (p. 238). He approached this capacity by way of the three 20th century philosophies of mathematics called logicism, formalism, and intuitionism. But he argued that "the usual view of them has suffered from an excessive preoccupation with quasi-technical 'philosophical'—or, perhaps better, ideological—issues and oppositions, in which perspective was lost of the mathematical interests these arose from" (p. 239). This lecture series explores the specific history of the mathematical interests that preceded the philosophies, to get deeper philosophical insights just as Stein suggested. Then we go on to later history including the past 75 years when new working methods rose first in structural mathematics and then in computing. These, rather than logicism, formalism, and intuitionism, are the methods that mathematicians use and debate today.

By a working method of mathematics I mean a way of finding and giving proofs, though of course this is not all that mathematicians do. By a philosophy here I mean an ontology or epistemology—a theory of what the things of mathematics are or of how we know them. I follow Saunders Mac Lane and John Mayberry in saying a foundation is a proposal for the organization of mathematics. It links proofs, ontology, and epistemology by proposing selected truths of mathematics as the base from which all theorems should be derived. This means foundations necessarily come after a considerable body of mathematics exists to be organized. While mathematical truths are intended to be timeless, foundations cannot be timeless.

Stein did not realize that the classification of mathematicians into logicians, formalists, and intuitionists originated just 20 miles from where he lived as he wrote his essay. It was done on August 28, 1893 by the tremendously influential German mathematician Felix Klein lecturing in English at Northwestern University outside the Chicago World's Fair. Klein spoke of these as working styles of specific leading mathematicians, and not as philosophies. And Klein specifies that a single mathematician may combine two styles. He calls himself an intuitionist and logician, while he calls Clebsch a formalist and intuitionist. So Klein had no need to argue that any one of them must work in practice, or should work in practice, or should work better than another. He just showed them working. And precisely because they were working, they changed over time. Each one changed its content and eventually changed from a working style into a philosophy, or, as Stein well says, an ideology.

In any field of thought, as a method achieves new results it normally grows in some ways and becomes more focused and specific in other ways. As the nineteenth century methods in mathematics grew they overlapped and interacted. By 1915 Emmy Noether became a leader in unifying working styles of mathematics that philosophers of mathematics today treat as rivals. This unification is standard procedure in mathematics today.

As Klein's distinction of working styles lost practical relevance, it hardened into the "three schools" of philosophy. In debate between Brouwer and Hilbert the terms shifted meanings. Brouwer essentially switched Klein's meanings of formalism and intuitionism. The term logician was replaced by a different word, *logicist*. Poincaré's ideas were interpreted after his death to depict him as an enemy of logic and somehow an early adherent to Brouwer's version of intuitionism. Philosophers who want to know how the philosophic debate between logicism, intuitionism, and formalism was resolved in practice, should be aware that the very terms of debate arose largely after the mathematical practice was settled.

Meanwhile new working methods arose in geometry and number theory. Contrary to much conventional wisdom these new methods led to vast unifications across mathematics and mathematical physics. The rise of electronic computing in the 1950s made other new developments possible. And all of this also led to still vaster unifying projects which have yet to be achieved.

The lectures trace these ideas historically, in part to illuminate them, but also to emphasize the flow of time. I urge you not only to adjudicate arguments and agree or disagree with any one or other view, or with my interpretations. Take philosophy seriously by asking what you would *do next*, either in mathematics or in philosophy, if you *believed* Dedekind, or Kronecker on concepts and calculations? or Poincaré, or Brouwer, or Hilbert on intuition? Or again if you believed Weil, or Grothendieck, Mac Lane, or Lawvere on the nature and role of structure in mathematics?