Epistemic Informativeness

Yanjing Wang* and Jie Fan

Abstract In this paper, we introduce and formalize the concept of *epistemic informativeness* (EI) of statements: the set of new propositions that an agent comes to know from the truthful announcement of the statements. We formalize EI in multi-agent Public Announcement Logic and characterize it by proving that two basic statements are the same in EI iff the logical equivalence of the two is common knowledge after a certain announcement. As a corollary applied to identity statements, $a = b$ and $a = a$ are different in EI iff $a = b$ is not common knowledge. This may shed new light on the differences in cognitive value of $a = a$ and $a = b$, even when they are both *known* to be true, as long as $a = b$ is not commonly known to all.

Key Words: cognitive value, epistemic informativeness, public announcement logic, common knowledge, Frege's puzzle

1 Introduction

Frege's puzzle of identity statements, in its simplest form, can be stated as the following question:

> How do we explain the difference between $a = a$ and $a = b$ in cognitive value to a linguistically competent speaker when a and b are co-referential?

To our view, there are at least three subproblems to be answered:

- What is the concept of 'cognitive value'?
- What is the concept of 'linguistic competence'?
- What is the proposition expressed by $a = b$ exactly?

Department of Philosophy, Peking University
e-mail: `y.wang@pku.edu.cn`, `fanjie@pku.edu.cn`

* corresponding author

Any solution to the puzzle should first sharpen the concept of cognitive value of statements. Instead of defining the cognitive value directly, many approaches in the literature attribute the *differences* in cognitive value to the differences in their *informativeness* (cf. e.g., Salmon (1986); Perry (1988); Yagisawa (1993)). Intuitively, $a = a$ and $a = b$ are indeed not equally informative, but what exactly is this concept of informativeness? How to compare the informativeness of two arbitrary statements? In this paper, we present a formal epistemic treatment of informativeness of propositional statements, which is not restricted to identity statements only, and prove technical results which may shed new light on the epistemic aspects of Frege's puzzle. Our discussion lies at the propositional level, and thus cannot fully account for Frege's puzzle, but we do hope to convince the readers that at this abstraction level, interesting things can be already said about phenomena related to the puzzle, when the knowledge of *multiple agents* is discussed formally. The rest of this section is devoted to informal ideas. Formal definitions and proofs will be given in the next section.

Our journey starts with the initial observation by Frege (1892): '$a = a$ holds *a priori* ... while statements of the form $a = b$ often contain very valuable extensions of our knowledge ...'. Intuitively, a statement is informative if it brings an extension to our knowledge. Thus it is crucial to understand what exactly is this extension. Note that the word 'extension' clearly suggests a *comparison* between the knowledge before and after the statement is made. Here, two questions arise naturally:

- Whose knowledge are we talking about in the comparison?
- How does the statement change knowledge?

In the previous work on Frege's puzzle, it is often implicitly assumed that there is a generic linguistically competent agent who is responsible for the informativeness of the statements. However, it is clear that a true statement ϕ may cause different knowledge extensions to different agents i and j, given that i knows ϕ already but j does not know it. Based on this observation, we should acknowledge the diversity of agents and their knowledge prior to the statement. Therefore, a reasonable definition of informativeness of a statement should be relative to a particular agent and also a concrete situation prior to the statement where the knowledge of the agent concerned is determined. The comparison is only possible if we fix these two factors.

To approach the second question, we first need to clarify how the agent concerned receives the statement. Our bottom line assumption is that the statement is communicated to the agent (from some source). In this paper, we fix the arguably simplest communication method: public announcement of the statement, i.e., the statement is announced publicly to all.[2] Now how does an announcement change the knowledge of the agents? Clearly, it is not as simple as adding the statement into the 'knowledge database'. For example, if you know $p \to q$ but not p nor q, then an announcement of p may let you know both p and q. On the other hand, the

[2] It is definitely possible to consider other communication methods which have different effects on the knowledge states of agents. The choice here is simple enough to make our points clear.

announced statement can even be false after the announcement, e.g., the announcement of a Moore sentence 'p and you do not know that p' will make itself false by letting the listener know p.[3]

Here we will follow the dynamic semantics tradition which dates back to the early works of Stalnaker (1978), Groenendijk and Stokhof (1991) and Veltman (1996), where the semantics of a statement is attributed to its *potential* in changing the common ground of the participants of the dialogue. The modern logical development of this idea is the framework of Dynamic Epistemic Logic (**DEL**) initiated by Plaza (1989) and Gerbrandy and Groeneveld (1997), where the communicative actions of statements are interpreted as transformers of knowledge states. We will make use of the simplest kind of **DEL**, the Public Announcement Logic (**PAL**) to formalize our concepts in the next section.

Now we are ready to state the informal definition of the central concept of this paper: *epistemic informativeness*.[4]

> The epistemic informativeness (EI) of a true statement to an agent given a concrete situation is the set of new propositions that the agent comes to know from the public announcement of the statement.

The adjective 'epistemic' suggests that this concept is a particular kind of informativeness: it is about knowledge but not belief which can be false,[5] nor the pragmatics induced information which comes along with the statements as discussed by Perry (1988). We claim that a difference in epistemic informativeness implies a difference in informativeness thus implies differences in cognitive value of statements, but the other way around may not hold, since the latter two concepts should be more general. Modest as it may look, epistemic informativeness can help us to explain non-trivial phenomena due to the fact that the knowledge in the definition is not only about basic facts but also about others' knowledge. As an example, let us consider the following strengthened version of Frege's puzzle:

> How do we explain the difference between $a = a$ and $a = b$ in cognitive value when they are both true and the agent already **knows** that $a = b$.

Here the multi-agent perspective is important. A public announcement of $a = b$ may not advance your knowledge about *basic facts* if $a = b$ is already known to you, but it may extend others' knowledge and this will in turn extend your knowledge about others' knowledge about $a = b$. Thus $a = b$ is still possibly informative even when the agent in concern knows it already. Actually, the situation is more subtle, for example, suppose that $a = b$ is known to *all* the agents, can it still be possibly different to $a = a$ in epistemic informativeness? More generally, when are

---

[3] Cf. Holliday and Icard (2010) for a detailed technical discussion of the Moore sentences in this setting.

[4] In the context of Frege's puzzle, a similar informal 'dynamic' conception of informativeness was briefly mentioned by Fiengo and May (2002), but not formalized precisely.

[5] False statements can also be informative in general if we are talking about belief.

two statements equally informative? Now we hit the boundary of the informal discussion and such issues will only be made clear using the formal tools in the next section.

In the formal part of the paper, we will do the following:

- We formalize the notion of epistemic informativeness of propositional statements and its conditional variant in Public Announcement Logic.
- We show that two basic statements are equally epistemically informative iff the logical equivalence of the two is commonly known after the announcement that one of them is true.
- As a consequence of the above result, (under two intuitive assumptions) if the proposition expressed by $a = b$ is not commonly known (no matter how close to common knowledge) then $a = b$ and $a = a$ are different in their epistemic informativeness.

At the end of the paper, we will come back to Frege's puzzle, and compare our approach to epistemic two-dimensionalism proposed by Chalmers (2004, 2011).

2 Formalization and proofs

We first review some basics about **PAL** (cf. e.g., van Ditmarsch et al (2007)). Given a set of agent names **I** and a set of proposition letters **P**, the language of **PAL** is defined as follows:

$$\phi ::= p \mid \neg\phi \mid (\phi \wedge \phi) \mid K_i\phi \mid C\phi \mid \langle\phi\rangle\phi$$

where $p \in \mathbf{P}$, and $i \in \mathbf{I}$. $K_i\phi$ expresses that 'agent i knows that ϕ' and $C\phi$ reads 'ϕ is common knowledge among all the agents'.[6] $\langle\psi\rangle\phi$ says that 'ψ can be truthfully announced, and after its announcement, ϕ holds.' As usual, we define $\phi \vee \psi$, $\phi \to \psi$, $\hat{K}_i\phi$, and $[\psi]\phi$ as the abbreviations of $\neg(\neg\phi \wedge \neg\psi)$, $\neg\phi \vee \psi$, $\neg K_i\neg\phi$, and $\neg\langle\psi\rangle\neg\phi$ respectively. In particular, $[\psi]\phi$ says that *if* ψ can be announced, then after its announcement ϕ holds.

The semantics of **PAL** is defined on S5 Kripke structures $\mathscr{M} = \langle W, \{\sim_i \mid i \in \mathbf{I}\}, V_{\mathbf{P}}\rangle$, where W is a non-empty set of possible states (epistemic possibilities), $\sim_i$ is an equivalence relation over W, $V_{\mathbf{P}} : W \to 2^{\mathbf{P}}$ is a valuation assigning each world a set of basic propositions which are true on it. An S5 model with a designated world w in it is called a pointed model (notation: $\mathscr{M}_w$). The semantics of **PAL** formulas is defined as follows:

[6] See van Ditmarsch et al (2007) for a discussion on common knowledge. Here common knowledge of ϕ means all the agents know ϕ and all the agents know that all the agents know ϕ, and so on, *ad infinitum*.

$$\begin{array}{|rcl|}\hline \mathcal{M}_w \vDash p & \Leftrightarrow & p \in V_{\mathbf{P}}(w) \\ \mathcal{M}_w \vDash \neg\phi & \Leftrightarrow & \mathcal{M}_w \nvDash \phi \\ \mathcal{M}_w \vDash \phi \wedge \psi & \Leftrightarrow & \mathcal{M}_w \vDash \phi \text{ and } \mathcal{M}_w \vDash \psi \\ \mathcal{M}_w \vDash K_i\psi & \Leftrightarrow & \text{for all } v \text{ such that } w \sim_i v : \mathcal{M}_v \vDash \psi \\ \mathcal{M}_w \vDash C\psi & \Leftrightarrow & \text{for all } v \text{ such that } w \sim^* v : \mathcal{M}_v \vDash \psi \\ \mathcal{M}_w \vDash \langle\psi\rangle\phi & \Leftrightarrow & \mathcal{M}_w \vDash \psi \text{ and } (\mathcal{M}|_\psi)_w \vDash \phi \\ \hline \end{array}$$

where $\sim^*$ is the transitive closure of $\bigcup_{i\in\mathbf{I}} \sim_i$ and $\mathcal{M}|_\psi = (W', \{\sim'_i \mid i \in \mathbf{I}\}, V'_{\mathbf{P}})$ where:

$$W' = \langle v \mid \mathcal{M}_v \vDash \psi\rangle, \sim'_i = \sim_i|_{W'\times W'}, V'_{\mathbf{P}} = V_{\mathbf{P}}|_{W'}$$

Intuitively, agents commonly know ϕ at w if ϕ holds in all the reachable states from w via $\sim^*$. An announcement $\langle\psi\rangle$ is interpreted as a *model transformer* which deletes the states that do not satisfy ψ. As usual, we write $\Delta \vDash \phi$ if ϕ semantically follows from the set of formulas Δ, i.e., for any $\mathcal{M}_w$ that satisfies all the formulas in Δ, $\mathcal{M}_w \vDash \phi$. The set of validities of **PAL** can be axiomatized by an extension of the proof system of S5 (see van Ditmarsch et al (2007) for details).

Clearly $\mathcal{M}_w$ can be viewed as a concrete scenario where each agent's knowledge is determined. Now, to formalize EI, we just need to collect the new knowledge χ that the agent learns after the announcement, i.e. $\mathcal{M}_w \vDash \neg K_i\chi \wedge \langle\phi\rangle K_i\chi$. Moreover, besides a single concrete scenario, it is also natural to discuss EI given a class of concrete scenarios given by some assumptions, e.g., we may want to compare EI of $a = b$ and $a = a$ given that $a = b$ and $K_i(a = b)$ are both true. This leads us to the concept of conditional EI formalized below with respect to a set Δ of **PAL** formulas. EI can be lifted naturally to a conditional notion by collecting all the pairs of a concrete scenario and the corresponding set of new knowledge.[7]

Definition 1. Given a pointed model $\mathcal{M}_w$ which satisfies ϕ, the epistemic informativeness (EI) of ϕ to agent i at $\mathcal{M}_w$ is the set (denoted as $ei(\phi, i, \mathcal{M}_w)$):

$$\{\chi \mid \mathcal{M}_w \vDash \neg K_i\chi \wedge \langle\phi\rangle K_i\chi\}$$

Based on this, we can define the conditional epistemic informativeness of ϕ to agent i given the assumption set Δ (denoted as $ei(\phi, i, \Delta)$) as the following set:

$$\{\langle\mathcal{M}_w, \Gamma\rangle \mid \mathcal{M}_w \vDash \Delta \cup \{\phi\} \text{ and } \Gamma = ei(\phi, i, \mathcal{M}_w)\}$$

Under this definition, we can naturally compare the EI of different statements by their corresponding knowledge sets. In the rest of this paper, we will focus on the equivalence of EI. We call a **PAL**-formula ϕ a *basic formula* if it does not contain modalities (neither K_i nor C nor any $[\psi]$). It is not hard to see that the truth value of a basic formula is preserved under announcements, since the truth value of a basic

[7] Rendsvig (2012) also used the update mechanism to define the *equality* of informativeness of identity statements, but in a very strict way: ϕ and ψ are equally informative if the corresponding announcements can delete the same states in the given model. Here we give a natural definition of informativeness itself, which induces a weaker notion of informational equivalence.

formula only depends on the valuation $V_{\mathbf{P}}$ which essentially stays the same after the announcement.

Now we are ready to prove our main result: the characterization of EI-equivalence given the assumption set Δ. At the first glance, the readers may think that we can simply use common knowledge of $\phi \leftrightarrow \psi$ to characterize the equality of ϕ and ψ in their informativeness. However, this does not work even when we restrict ourselves to very simple propositional formulas. As an example, consider the following pointed model $\mathscr{M}, w$ (reflexive arrows omitted):

$$w : p, q \overset{1}{\longleftrightarrow} \neg p \neg q \overset{2}{\longleftrightarrow} p, \neg q$$

Clearly, updating p or q cause the same effect, i.e., $ei(p, 1, \mathscr{M}_w) = ei(q, 1, \mathscr{M}_w)$, but $\mathscr{M}_w \not\vDash C(p \leftrightarrow q)$. Actually, the characterization result is more subtle, and that is why we need a proper technical treatment:

Theorem 1. *For any set Δ of **PAL** formulas and any basic **PAL** formulas ϕ, ψ:*

$$ei(\phi, i, \Delta) = ei(\psi, i, \Delta) \iff \Delta \vDash [\phi \vee \psi] C(\phi \leftrightarrow \psi).$$

Proof. $\Leftarrow$: Suppose that $\Delta \vDash [\phi \vee \psi] C(\phi \leftrightarrow \psi)$. Now take an arbitrary $\mathscr{M}_w$ such that $\mathscr{M}_w \vDash \Delta$. Suppose that $\mathscr{M}_w \vDash \neg K_i \chi \wedge \langle \phi \rangle K_i \chi$ for some χ, we need to show $\mathscr{M}_w \vDash \neg K_i \chi \wedge \langle \psi \rangle K_i \chi$ which amounts to show $\mathscr{M}_w \vDash \psi$ and $(\mathscr{M}|_\psi)_w \vDash K_i \chi$. Now since $\mathscr{M}_w \vDash \Delta$ and $\mathscr{M}_w \vDash \phi$ (due to $\mathscr{M}_w \vDash \langle \phi \rangle K_i \chi$), we have $\mathscr{M}_w \vDash \phi \wedge [\phi \vee \psi] C(\phi \leftrightarrow \psi)$ which means $\phi \leftrightarrow \psi$ holds on all the w-reachable states in $\mathscr{M}|_{\phi \vee \psi}$. Now since w is reachable from w itself in $\mathscr{M}|_{\phi \vee \psi}$ (due to the fact that $\mathscr{M}$ and $\mathscr{M}|_{\phi \vee \psi}$ are S5 models), $(\mathscr{M}|_{\phi \vee \psi})_w \vDash \phi \leftrightarrow \psi$. Since $\mathscr{M}_w \vDash \phi$ and ϕ is basic, we have $(\mathscr{M}|_{\phi \vee \psi})_w \vDash \phi$, thus $(\mathscr{M}|_{\phi \vee \psi})_w \vDash \psi$ and $\mathscr{M}_w \vDash \psi$ for ψ is basic too. Now we show the w-reachable states in $\mathscr{M}|_\phi$ and $\mathscr{M}|_\psi$ are exactly the same. Suppose that v is reachable from w in $\mathscr{M}|_\phi$ via a path pa then pa should consist of ϕ-states only. Clearly, pa should also be in $\mathscr{M}_{\phi \vee \psi}$. Now since $\mathscr{M}_w \vDash \phi \wedge [\phi \vee \psi] C(\phi \leftrightarrow \psi)$, all the states in pa should also satisfy ψ which means pa is also in $\mathscr{M}|_\psi$. Therefore v is reachable from w in $\mathscr{M}|_\psi$. Similarly we can show that all the w-reachable states in $\mathscr{M}|_\psi$ are those reachable in $\mathscr{M}|_\phi$ which means the w-reachable parts in the $\mathscr{M}|_\phi$ and $\mathscr{M}|_\psi$ are isomorphic. Now it is clear that $(\mathscr{M}|_\psi)_w \vDash K_i \chi$. Till now we have shown $\chi \in ei(\phi, i, \mathscr{M}_w) \implies \chi \in ei(\psi, i, \mathscr{M}_w)$. The other implication is symmetric. Due to the arbitrariness of $\mathscr{M}_w$, we have $ei(\phi, i, \Delta) = ei(\psi, i, \Delta)$.

$\Rightarrow$: Towards contradiction, suppose that $ei(\phi, i, \Delta) = ei(\psi, i, \Delta)$ but $\Delta \nvDash [\phi \vee \psi] C(\phi \leftrightarrow \psi)$. Then there is a pointed model $\mathscr{M}_w$ such that $\mathscr{M}_w \vDash \Delta \cup \{\phi \vee \psi\}$, but $(\mathscr{M}|_{\phi \vee \psi})_w \nvDash C(\phi \leftrightarrow \psi)$. Thus by the semantics of the common knowledge operator C, there is a finite path from w to a state v in $\mathscr{M}|_{\phi \vee \psi}$ such that $(\mathscr{M}|_{\phi \vee \psi})_v \nvDash \phi \leftrightarrow \psi$. Suppose that pa is (one of) the shortest such paths then all the states preceding v in pa satisfy $\phi \leftrightarrow \psi$. Since pa is in $\mathscr{M}|_{\phi \vee \psi}$ and it is the shortest, all the states preceding v in pa satisfy $\phi \wedge \psi$ $(\star)$. Now W.L.O.G. we assume $(\mathscr{M}|_{\phi \vee \psi})_v \vDash \phi \wedge \neg \psi$ thus $\mathscr{M}_v \vDash \phi \wedge \neg \psi$ since ϕ and ψ are basic. Since $ei(\phi, i, \Delta) = ei(\psi, i, \Delta)$, it is not hard to see that $\mathscr{M}_w \vDash \phi \leftrightarrow \psi$. Since $\mathscr{M}_w \vDash \Delta \cup \{\phi \vee \psi\}$, we know $\mathscr{M}_w \vDash \phi \wedge \psi$. Now let χ be $C\psi$, it is clear that $\mathscr{M}_w \vDash \neg K_i \chi$ and $\mathscr{M}_w \vDash \langle \psi \rangle K_i \chi$. However, $\mathscr{M}_w \nvDash \langle \phi \rangle K_i \chi$

since $\mathscr{M}_v \nvDash \phi \leftrightarrow \psi$ and the path pa from w to v is preserved in $\mathscr{M}|_\phi$ due to the fact $(\star)$.

In words, the above theorem says that given an assumption, two basic formulas ϕ and ψ have the same EI if and only if the logical equivalence of the two is common knowledge after its announced that one of them is true. Note that $\phi \leftrightarrow \psi$ does not need to be common knowledge to equate the EI of ϕ and ψ.[8] Also note that agent i disappeared at the right hand side of the equivalence stated in the above theorem, which suggests that without mentioning the agent in question, we can also meaningfully talk about the EI-equivalence of two statements given the same assumption. In fact, since $C\chi \leftrightarrow K_iC\chi$ is valid for any $i \in \mathbf{I}$ and $\chi \in \mathbf{PAL}$, the agent i is implicitly there.

Let $\Delta_n^\psi = \{K_{i_1}K_{i_2}\cdots K_{i_k}\psi \mid k \leq n, i_h \in \mathbf{I}\}$, in particular $\Delta_0^\psi = \{\psi\}$ then the following corollary is immediate, based on the observation that if ϕ is valid then $\phi \vee \psi$ is valid and $\phi \leftrightarrow \psi$ is equivalent to ψ.

Corollary 1. *For any basic **PAL** formulas ϕ and ψ, any set of **PAL** formulas Δ, if ϕ is valid then:*

1. $ei(\phi, i, \Delta) = ei(\psi, i, \Delta) \iff \Delta \vDash C\psi$
2. $ei(\phi, i, \Delta_n^\psi) \neq ei(\psi, i, \Delta_n^\psi)$ for all $n \in \mathbb{N}$.

Note that (2) follows from (1) since $\Delta_n^\psi \nvDash C\psi$ for any $n \in \mathbb{N}$. This shows that unless ψ is common knowledge under the current assumption, it is different to a tautology in EI.

Finally, let us get back to the strengthened version of Frege's puzzle mentioned before, where we are supposed to explain the differences in cognitive value of $a = a$ and $a = b$ even when $a = b$ is known to the agents. To apply the above corollary, let us first make two 'innocent' assumptions:

A $\quad a = b$ and $a = a$ express some propositions, no matter what they are.
B $\quad$ the proposition expressed by $a = a$ is valid.

Under (A) and (B), Corollary 1 tells us that given any Δ, $a = a$ and $a = b$ have different EI unless the proposition expressed by $a = b$ is common knowledge. Note that, under non-trivial a and b, such as Hesperus and Phosphorus, the common knowledge of the proposition expressed by $a = b$ seems to be very hard to obtain, given the set of agents including *all* the people in the world. Therefore $a = b$ is almost always different from $a = a$ in EI of their corresponding propositions, which may help to explain our intuition of the differences in cognitive value between two statements.

[8] Actually, $[\phi \vee \psi]C(\phi \leftrightarrow \psi)$ can be formulated as a weaker version of common knowledge of $\phi \leftrightarrow \psi$, which is called relativized common knowledge (cf. van Benthem et al (2006)).

3 Conclusion

We introduced the formal concept of epistemic informativeness based on a multi-agent dynamic epistemic perspective, and characterized its equivalence by using common knowledge after an announcement. We suggest that the lack of common knowledge of the logical equivalence of two statements is (partly) responsible for the difference in cognitive value of the two. Clearly, similar analysis can be done with respect to belief and communication methods other than public announcements.

Coming back to Frege's puzzle, as we claimed, epistemic informativeness can be used to explain the differences in cognitive value. This epistemic account for cognitive value can be compared to the theory of epistemic two-dimensional semantics (ETDS) proposed by Chalmers (2004), where *epistemic intension* plays a similar role as EI and the *scenarios* are like the possible states in our epistemic models. According to ETDS, two sentences that share the same subjunctive intension (secondary intension) may still differ in cognitive value due to differences in epistemic intension (primary intension). However, there is a fundamental difference between epistemic intension and EI: epistemic intension of a sentence is a function mapping scenarios to extensions (simply `ture` or `false`) while EI of a sentence can be viewed as a function mapping multi-agent epistemic pointed models to collections of new knowledge. It seems that EI is a more refined candidate for the Fregean 'sense', although in the current formal rendering it also shares some of the drawbacks that epistemic intension suffers, e.g., the current formal account of EI cannot explain the differences in cognitive value of different tautologies, since tautologies are taken to be common knowledge due to logical omniscience of the underlying epistemic framework.[9] To obtain a more reasonable formal account, we may select the appropriate epistemic framework modelling bounded reasoning power.

As the readers must have noticed, our formal discussion is purely propositional, thus we cannot formalize the exact proposition expressed by equalities such as Phosphorus=Hesperus. Moreover, this propositional abstraction level does not facilitate us to formally talk about the linguistic competence of the agents, for which we do need extra epistemic operators expressing 'I know what Hesperus is/means', as discussed in Wang and Fan (2013). To really compare with ETDS which reconciles apriority and necessity, we also need to talk about necessity and interpret it with respect to metaphysically possible worlds. Nevertheless, separating different issues relevant in Frege's puzzle may give us a clearer picture of the merit of the puzzle and its solutions. We leave an in-depth formal study to Frege's puzzle based on EI to a further occasion.

[9] For example, '$2^{57885161} - 1$ is a prime number' may have the same EI as '2 is a prime number', although intuitively these two sentences should induce different knowledge updates if not all the tautologies are common knowledge.

Acknowledgements We are grateful to Wen-fang Wang for his insightful comments on an earlier version of this paper. Yanjing Wang acknowledges support of the key project 12&ZD119 of the National Social Science Foundation of China.

References

van Benthem J, van Eijck J, Kooi B (2006) Logics of communication and change. Information and Computation 204(11):1620–1662

Chalmers DJ (2004) Epistemic two-dimensional semantics. Philosophical Studies 118:153–226

Chalmers DJ (2011) The nature of epistemic space. In: Egan A, Weatherson B (eds) Epistemic Modality, Oxford University Press, pp 1–54

van Ditmarsch H, van der Hoek W, Kooi B (2007) Dynamic Epistemic Logic. Springer

Fiengo R, May R (2002) Identity statements. In: Logical Form and Language, Clarendon Press, pp 169–203

Frege G (1892) Über Sinn und Bedeutung. Zeitschrift für Philosophie und philosophische Kritik 100:25–50

Gerbrandy J, Groeneveld W (1997) Reasoning about information change. Journal of Logic, Language and Information 6(2):147–169

Groenendijk J, Stokhof M (1991) Dynamic predicate logic. Linguistics and Philosophy 14(1):39 – 100

Holliday WH, Icard TF (2010) Moorean phenomena in epistemic logic. In: Advances in Modal Logic, pp 178–199

Perry J (1988) Cognitive significance and new theories of reference. Noûs 22(1):1–18

Plaza JA (1989) Logics of public communications. In: Emrich ML, Pfeifer MS, Hadzikadic M, Ras ZW (eds) Proceedings of the 4th International Symposium on Methodologies for Intelligent Systems, pp 201–216

Rendsvig RK (2012) Modeling semantic competence: a critical review of freges puzzle about identity. New Directions in Logic, Language, and Computation pp 152–162

Salmon N (1986) Frege's Puzzle. The MIT Press

Stalnaker R (1978) Assertion. In: Cole P (ed) Syntax and Semantics, vol 9, New York Academic Press

Veltman F (1996) Defaults in update semantics. Journal of Philosophical Logic 25(3):221–261

Wang Y, Fan J (2013) Knowing that, knowing what, and public communication: Public announcement logic with `Kv` operators. In: Proceedings of IJCAI, pp 1139–1146

Yagisawa T (1993) A semantic solution to frege's puzzle. Philosophical Perspectives 7:135–154