Johan van Benthem, **Logical Dynamics of Information and Interaction**, Cambridge University Press, 2011, pp. 386. ISBN: 9780521765794 (hardback) GBP: £55

1 Background and intended audience

The book *Logical Dynamics of Information and Interaction* is the third monograph of Johan van Benthem dedicated to the 'dynamic turn' of logic centering on the idea of making inference and actions of language use first-class citizens of logical theory. Compared to its predecessors *Language in Action* (1991) and *Exploring Logical Dynamics* (1996), this 2011 book presents an evolved view of the author on logical dynamics: it is about the intelligent agents who produce, transform and convey information in their social interactions. The book presents a systematic study based on the semantics-driven logical framework of dynamic-epistemic logic (DEL) developed around 2000. The author's upcoming new book *Logic and Games* (2013) will elaborate more on logical dynamics in games discussed briefly in Chapters 10 and 15 of the current book.

This book is not a textbook and, despite the innocent looking due to the cheerful van Benthem style, it requires a good knowledge about modal logic: at least at the level of the basic tracks in *Modal Logic* (2001) by Blackburn, de Rijke and Venema. Moreover, the readers may also like to prepare themselves by going through the core theory of *Dynamic Epistemic Logic* (2007) by van Ditmarsch, van der Hoek, and Kooi, and also van Benthem's own textbook *Modal Logic for Open Minds* (2010) which has chapters dedicated to various dynamic-epistemic logics with more technical details.

This book is a rich collection of recent developments in the field of DEL (in a broad sense) with insightful connections to related fields, accompanied by many concrete examples. It is a must-read for the researchers and advanced students in the field, for the valuable general picture of the state of the art that it provides. Although the author tried to be selective in composing the book (as mentioned in the preface), some readers may still share my feeling: the information on each topic that the book touches is simply overwhelming and it touches many different threads. The author has poured so many thoughts into this book, which one may sense from the numerous lengthy footnotes and pages of discussions after the conclusion section in each chapter. The down side of this is that one may get lost in these adventures and miss some important points and links here and there. On the other hand, if you skip all these subtleties and excursions, the book seems much easier to follow for there are mainly examples, facts, remarks, observations and analogies one after another but not so many detailed proofs. However, the impression that an inexperienced reader may have by casually browsing over such a book can be misleading. I would advise the serious readers to really work out the details omitted in the book and check the relevant references.[1] For others who just want to get a taste of this new ambitious research agenda in logic, it is fun to be a bit careless, but please make sure to warn yourself of the depth behind this colourful appearance.

In the rest of this review I will first try to streamline the core ideas of the book in Section 2 and summarize the structure of the book in Section 3. Then in Section 4 I will make some implicit ideas buried in between the chapters more explicit from a technical point of view. Finally I conclude with some latest developments which answered some of the open questions in this book. In the sequel, I will use 'the author' to refer to the author of the book.

[1]Note that some papers mentioned in this book are unpublished notes by the author which are hard to find. In such cases please contact the author of the book.

2 The basic ideas

The nature of this research is mainly a *normative* one: describing how *information* flows among *rational* agents in some ideal sense.[2] Thus there are two central concepts: rationality and information. *Rationality* in this book amounts to intelligently obtaining and using information. Depending on the levels of informational processes, being *rational* in this book has a layered meaning: to *reason* intelligently, to *act* intelligently, and to *interact* intelligently balancing information and goals of each other (cf. Ch. 1.2).[3] Here relevant actions include processes that obtain, correct, and use information such as observation, revision, and decision. *Information* in this book also accommodates diversity (cf. Ch. 13.3 for further discussion). In most of the book, the author seems to take information as a semantic range of possibilities, which forms knowledge, belief, and other attitudes of the agents. Based on these attitudes, agents evaluate their actions, make decisions and interact with each other: not only one shot actions but also long term plans. The goal of the book is to find logical laws in these actions.

The book employs a highly uniform yet flexible technical methodology from DEL, which involves a semantic tandem schema: *representation and transformation* (cf. pp. 332) plus the *recursion axiom schemata* for complete axiomatizations. In a nutshell, the methodology is to dynamify semantics-driven epistemic-like logics, which is summarized as follows (cf. Ch. 3.5):

> Start out with a static logic of a certain modal attitude of agents, interpreted on models *representing* the corresponding information in concern. Then *dynamify* the static logic by adding explicit action operators to the static logical language and correspondingly interpreting them as suitable model *transformers*. In this way, each model can be viewed as a snap shot of an evolving system driven by the transformers. If the static logic is expressive enough then we may use *recursion axiom* schemata to *reduce* the new logic into the old, static one qua expressive power, thus also obtaining an axiomatization of the new logic. These recursion axiom schemata demonstrate a distinct feature of the DEL approach: it analyses post-conditions *compositionally*. The desired reduction may require a redesign of the static language in order to have enough *pre-encoding power* while still balancing the expressive power and the computational complexity.

This methodology has been carried out in most of the book when reductions are possible. However, as the author puts it on pp. 62, 'there are better reasons for relaxing the orthodox [reduction-based] version' of this methodology (cf. Ch. 11). In particular, note the difference between the terminology 'recursion axioms' vs. the more commonly used 'reduction axioms'. I will address the story behind it in section 4 as one of the hidden treasures in this book.

3 The structure of the book

Ch. 1 explains the objective of the study and presents an overview of the book. The remaining chapters, except the conclusion (Ch. 17), can be divided into three parts: (I) the core theory (Ch. 2-4), (II) aspects of logical dynamics (Ch. 3-12), and (III) connections to other fields (Ch. 13-16).

In Part (I), Ch. 2 first reviews the basics of classic epistemic logic from a model-theoretical point of view, and lays out the main technical schema of the study: axiomatization, invariance,

[2] Also see the discussion on the connection to cognitive reality in Ch. 16.

[3] In Ch. 13.5 the author refines the concept by giving four features: taking and combining information intelligently, self-correcting intelligently, balancing between information, preference and goals intelligently, and interacting intelligently.

expressive power, and computational complexity. Ch. 3 introduces the epistemic framework of public observation (known as the *public announcement logic* viz. PAL) and the dynamic methodology throughout the book. Ch. 4 then generalizes the framework to handle dyanmic-epistemic reasoning with partial observation where new possibilities may be generated (known as the event-model based *dynamic-epistemic logic*).

Part (II) can be further divided into two sub-parts: (IIa: Ch. 3-9) variants of the core theory on other modal attitudes of the agents, and (IIb: Ch. 10-12) direct applications to related fields. Each chapter in (IIa) can be viewed as a miniature of Part (I) in a different setting with the following general structure: (1) philosophical background with running examples; (2) basic models and the static language; (3) some model theoretical results about the static logic; (4) model transformers and axiomatization of the dynamified logic via recursion axiom schemata; (5) applications and conclusions; (6) further related issues; (7) a brief note on related literature. The chapters in (IIb) roughly follow the above structure too, although they concentrate on the direct applications of the core theory to game theory, epistemic temporal logic, and social choice theory. As the author remarked, these three chapters present the logical dynamics of long-term interaction. A rough summary of Part (II) is as follows (Ch. 3 and 4 are included for comparison).

chapter	representation	transformation
3 PAL	epistemic model (EM)	relativization
4 DEL	EM	product update
5 awareness	EM + accessible sets	relativization and realization
6 issue management	EM + issue relations	link-intersection and product update
7 belief	EM + plausibility relations	lexicographic/conservative upgrades
8 probability	EM + probability distributions	probabilistic product update
9 preference	modal betterness model	like Ch.7, defined by PDL programs
10 games	EM + moves (extensive games)	relativization and product update
11 procedures	EM + protocols	relativization and product update
12 groups	doxastic model	priority update

Part (III) connects logical dynamics to philosophy (Ch. 13) computer science (Ch. 14), game theory (Ch. 15) and cognitive science (Ch. 16), by showing several concrete examples besides general philosophical remarks. In particular, Ch. 15 differs from Ch. 10 by focusing on strategic games as epistemic models over strategy profiles, instead of extended form games with imperfect information discussed in Ch. 10. As the author remarked, by dynamic-epistemify the game solutions we can see more clearly what makes it tick in game theory.

4 A guide for hidden treasures and unsaid notes

Even for a researcher in the field, the information contained in this book is overwhelming. To navigate in this spring of information is no easy task, let alone the pleasant distractions you may get here and there in the unique van Benthem style. Sometimes, important things also appeared as footnotes or remarks after the sections of conclusion, which may slip away from the eyes of the reader. In the following, by showing my personal picks for the hidden treasures and unsaid notes in the core technical theory of the book, I hope to navigate the serious readers (researchers in the field) some more to appreciate the merit of the book and also the technical subtlety of the field.[4]

[4]The results and remarks are not numbered in this book, thus I will always mention the page numbers in the following discussion.

Let me start from a term which plays an important role in the book. Readers who are familiar with DEL may notice that the author uses the term 'recursion axioms' (pp. 53) instead of the commonly used 'reduction axioms'.[5] The idea behind this switch is important, though not explained explicitly in the book.[6] The term originates from the *recursive* definition of the syntactic relativization of modal logic which corresponds to the standard axioms of PAL (cf. pp. 73: *The secret of PAL*). More importantly, the term 'reduction axioms' may suggest that the reduction to the static logic is the only purpose, while the author of the book believes that the reduction may well be a side effect in some cases and we may axiomatize the DEL-like logics without a reduction, as demonstrated in Ch. 11 of the book (cf. also pp. 62: *Relaxing the reduction programme*).[7] Finally, by using the term the author also wants to draw the analogy between the recursive axioms and the differential equations used in dynamical systems, which may fit logical dynamics in a bigger picture (cf. pp. 98). It is an interesting perspective to be made precise.

Having said the above, the reduction technique is still crucial in this book, and the (effective) closure of a logic w.r.t. model transformers is definitely a desired property (cf. pp. 93). However, reduction also suggests *redundancy*, which should be discussed by any book on DEL. Besides the arguments against the redundancy view on pp. 61, there is also a solid technical argument hidden in Footnote 31 on pp. 70, which should be emphasized: PAL is *exponentially more succinct* than EL but the complexity of the satisfiability problems of the two are equally hard as showed in [5, 2]. Therefore PAL (and in fact many other DEL logics) does gain the advantage in succinctness without any loss at the complexity of the logic. Note that this is not a counter example to the *golden rule* mentioned on pp. 38 between the *expressive power* and the computational complexity.

Let us continue our journey of the axioms by moving to Footnote 8 (pp. 53) which states that in the axiomatizations throughout the book, the necessitation rule (for the update operators) and the replacement of equals are assumed. This is crucial since the reduction-to-static argument in this book needs to make use of the later rule, unless the composition axioms (cf. e.g., pp. 57 and pp. 98), which combine two updates into one, are included in the system (cf. [7] for detailed discussion).[8] Note that, some of the logics in this book do not validate composition axioms (e.g., pp. 119: FACT). The readers are recommended to workout the details of the reductions in order to understand the importance of those rules.

Another topic about the recursion axioms that should receive more attention is the use of them as abstract postulates of updates (cf. Section 3.8 for relativization and page pp. 152 on lexicographic upgrade). While using axioms to characterize frame properties is a standard exercise in modal logic, using axioms to characterize updates (relations on the 'update universe') is not yet well studied (see Footnote 35 on pp. 151 for subtlety between the two). The new correspondence theory can help us to understand the axiomatization of the dynamic-epistemic logics better. A related interesting issue is the preservation of frame

[5] Another terminology switch is from the commonly used 'action models' to the more neutral 'event models' (pp. 79), since events may include observables which are not really actions, such as a rain or an accident.

[6] The following explanation is due to personal correspondence with the author of the book.

[7] The TPAL introduced in Ch. 11 cannot be reduced to epistemic logic since the temporal development of the system are not computable from the purely epistemic information due to the extra protocol information. On the other hand, TPAL can be reduced to the extension of epistemic logic with $\langle\varphi\rangle\top$ formulas (Footnote 23 on pp. 243). Also note that the axiom $\langle\varphi\rangle\top \to \varphi$ should be added to the system on pp. 243 in order to obtain completeness (cf. [6]).

[8] Actually including the necessitation rule and the K axioms for update operators can make sure that the replacement of equals is an admissible rule (cf. also [7]).

conditions by various transformers (e.g., cf. Footnote 20 on pp. 60, and also pp. 183).

To end this excursion on axioms, we move to Ch. 11 which should, in my view, belong to the core theory due to its importance in viewing the dynamics in the update universe. It also links DEL to the related (if not competing) approach of modelling logical dynamics: the epistemic temporal logic (ETL). The author abstracts the recursion axiom w.r.t the knowledge operator into the schema $K[e]\psi \leftrightarrow [e]K\psi$ which stipulates two assumptions of the agents: the $\rightarrow$ part is the *perfect recall* axiom schema and the $\leftarrow$ is the *no learning* axiom schema in ETL (cf. pp. 57).[9] The discussion continues in Ch. 11 on pp. 231 about the two corresponding temporal epistemic properties. However, it is crucial to note that no learning is *not* valid in dynamic-epistemic logics in general, e.g., the PAL formula $[p]K\neg p \rightarrow K[p]\neg p$ does not hold on the left world of the model $\neg p$ —— p. Actually, given $(p \rightarrow [e]p) \wedge (\neg p \rightarrow [e]\neg p)$ (propositional invariance) and no learning schema on top of S5, we can derive $\langle e\rangle Kp \rightarrow Kp$ which literately says agents cannot learn any new facts. Now, to match the author's remark right after the second Fact on pp. 231, we need a slightly different version: $\langle e\rangle K\psi \rightarrow K[e]\psi$ (*no miracles*) which says exactly that people cannot learn *if* actually observing e, while leaving it *possible* that they may learn something if e is not observed. We can state the corresponding properties more accurately as: for all ke and h, if $k \sim h$ **and** he exists, then $ke \sim he$. This subtle and almost neglectable difference between no learning and no miracles may carry a secret of logical dynamics: learning by observing.[10] Actually such axiom schemata (no miracle, perfect recall, and propositional invariance) can be used to axiomatize the logical dynamics *without* using the reduction techniques (cf. [7]).

To continue like this will take the whole issue of the journal, therefore let me just pick up some of the nice results mostly hidden in footnotes and remarks in the rest of the book.

(Compositionality, again) As we mentioned, the methodology in the book features a compositional analysis of the post-conditions via recursion axioms. This compositionality matters a lot when complicated post-conditions are in concern, as demonstrated by several instances scattering across the book: the failure of AGM postulates in DEL (cf. pp. 150 and Footnote 32 on the same page); the failure of Bayes' rule in DEL (cf. pp. 162); and finally the analysis of the knower's paradox on pp. 269.

(Dynamic inference) This is a somehow forgotten topic which dates from the birth of dynamic semantics and DEL. It is good to recall the nice axiomatization result of the structural properties of the dynamic inference on pp. 69 and in Footnote 30 on pp. 299. As the author remarked on pp. 301, the monotonic dynamic-epistemic logics can surprisingly model non-monotonic inferences.

(On finite models) There are nice results about special cases when a finite model is given, e.g., the existence of persistent equivalents of any announcement (Footnote 65 on pp. 69), the definability result of safe belief on pp. 138, and the procedure to reach the communication core on pp. 256.

[9] The $\rightarrow$ direction says: If you know that observing e makes sure ψ then you will know ψ after observing e (thus *no forgetting* or, say, *perfect recall*). The $\leftarrow$ direction says If you know ψ after observing e then you know it already before hand (thus no learning).

[10] In the setting of the traditional ETL such as [3], the difference is indeed neglectable since instead of $[e]$, the next time operator O is used and there is no difference between its box-form and diamond-form for there is always a (unique) next time in linear temporal logic. In this light, we may need to be more careful in using the existing results of ETL on pp. 235. Since a slightly different property may cause a significant change in complexity.

(A general result about recursion axioms) The author talked about numerous recursion axiom schemata case by case, which may invite the reader to wonder about general results. There is one of this type in the book: for each PDL transformer there is a set of complete recursion axiom schemata on pp. 145.

(Iterations) The book also touches the iteration of events here and there and maybe the most important result besides the undecidability result (on pp.232) is the decidability of the PAL logic with limit announcement operator based on existential formulas on pp. 324 (see Ch. 15.5 for a detailed discussion).

There are also many technical results that I was not aware of, such as the characterization theorem for preference aggregation on pp. 205. Besides these technical things, the book is full of philosophical remarks and analogies which are thought-provoking. In particular, Ch. 13 shows the true heart of the author in philosophy. I am sure every reader at any level of familiarity to the field is bound to find something new and interesting in this book.

5 Further information and conclusion

The field of logical dynamics based on DEL framework is under rapid development. Some open problems mentioned in the book have been solved after the publication of the book in 2011. Here are just two examples:

problem	page	reference	remarks
axiomatization of the substitution core	74	[4]	infinitely many agents
computational complexity of DEL	96	[1]	NEXPTIME for satisfiability

There are also more technical open problems in the book which are not solved, such as the axiomatization of the logic over finite games on pp. 221. However, the readers may also be in favour of the numerous conceptual 'open problems'. Many of them are under progress. One may also find some new developments at the supporting website of the book: `http://www.illc.uva.nl/lgc/ldii/`.

Of course, there are more general questions to be asked about the study of logical dynamics presented in the book besides the technical ones. For example, one may want a more thorough comparison between the ETL approach and the DEL approach in handling the same logical dynamics: What are the Cons and Pros for each approach? Perhaps a more streamlined picture of future directions is also preferable besides the diversity in numerous applications. From a technical point of view, the core theory of logical dynamics via DEL still needs to be sharpened and deepened to fertilize important general results. I hope those thousand threads in the book will gravitate towards some central point in the future and that may mark the maturity of the field.

In sum, this book is the best we can have for now as a great source for the research in the field of logical dynamics of information and interaction. It can be used as a handbook of DEL as well. I think the author has succeeded in demonstrating a new view of logic as a theory of information flow in the interaction of agents. As argued in this book, logic can be more than it appears in a open mind (cf. pp. 62). In the end, some readers may share my experience: sometimes your best ideas are from others, in particular they may be hidden in Johan's early notes which you did not realize when browsing over them for the first time. In this light, make sure you read the book carefully and come back to it again (and again).

References

[1] Guillaume Aucher and Franois Schwarzentruber. On the complexity of dynamic epistemic logic. In *Proceedings of TARK '13*. ACM Digital Library, 2013.

[2] Tim French, Wiebe van der Hoek, Petar Iliev, and Barteld P. Kooi. Succinctness of epistemic languages. In *IJCAI*, pages 881–886, 2011.

[3] Joseph Y. Halpern and Moshe Y. Vardi. The complexity of reasoning about knowledge and time. i. lower bounds. *Journal of Computer and System Sciences*, 38(1):195 – 237, 1989.

[4] Wesley Holliday, Tomohiro Hoshi, and Thomas Icard. A uniform logic of information dynamics. In *Proceedings of Advances in Modal Logic 2012*, volume 9, pages 348–367. College Publications, 2012.

[5] Carsten Lutz. Complexity and succinctness of public announcement logic. In *Proceedings of AAMAS '06*, pages 137–143, New York, NY, USA, 2006. ACM.

[6] J. van Benthem, J. Gerbrandy, T. Hoshi, and E Pacuit. Merging frameworks for interaction. *Journal of Philosophical Logic*, 38(5):491–526, October 2009.

[7] Yanjing Wang and Qinxiang Cao. On axiomatizations of public announcement logic. *Synthese*, 190:103–134, 2013.

Yanjing Wang
Department of Philosophy
Peking University
100871 Beijing, CHINA
y.wang@pku.edu.cn