BEYOND "KNOWING THAT" (I)

A NEW GENERATION OF EPISTEMIC LOGICS

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NASSLLI 2018, CMU

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Background

About the course

Background

IMPORTANCE OF REASONING ABOUT KNOWLEDGE (IN EVERYDAY LIFE)

We use knowledge, belief and probability to organize certainty and uncertainty, and to turn uncertainty into certainty.

- Knowledge is power: act properly to achieve goals;
- Knowledge is time: to make decisions more efficiently;
- Knowledge is money: can be traded;
- Knowledge is responsibility: to prove someone is guilty;
- Knowledge is you: to identify oneself;
- Knowledge is an immune system: to protect you;
- Knowledge satisfies our curiosity

Reasoning about knowledge: *know the unknown from the known* (and new info).

STANDARD EPISTEMIC LOGIC

Modal logics that reason about (propositional) knowledge (and belief) [von Wright 1951, Hintikka 1962]

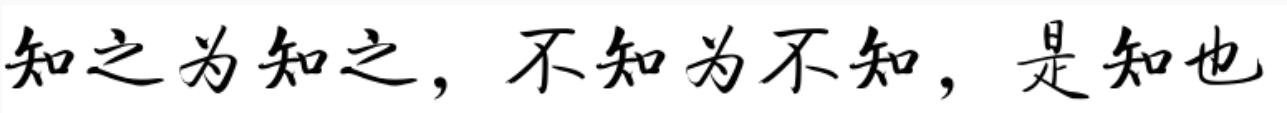
- Language: "agent *i knows that* φ" ($K_i\varphi$).
- Model: "possible worlds" with epistemic relations
- Semantics: you know that φ iff φ is true in all the epistemic alternatives that you cannot distinguish from the actual world.

$$s \vDash p \wedge \neg K_i p$$

S5 system (strongest epistemic logic)

Axioms		Rules	
TAUT	all the instances of tautologies	MP	$\dfrac{\varphi, \varphi \to \psi}{\psi}$
DISTK	$\mathcal{K}_i(p \to q) \to (\mathcal{K}_i p \to \mathcal{K}_i q)$	NECK	$\dfrac{\varphi}{K_i\varphi}$
T	$\mathcal{K}_i p \to p$	SUB	$\dfrac{\varphi}{\varphi[p/\psi]}$
4	$\mathcal{K}_i p \to \mathcal{K}_i \mathcal{K}_i p$		
5	$\neg \mathcal{K}_i p \to \mathcal{K}_i \neg \mathcal{K}_i p$		

Analogues of axioms 4 and 5 in Confucius' teaching:



Powerful when combined with other modalities: ETL, DEL etc.

BEYOND "KNOWING THAT"

Knowledge is not only expressed in terms of "knowing that":

- I *know whether* the claim is true.
- I *know what* your password is.
- I *know how* to go to the hotel.
- I *know why* he was late.
- I *know who* proved this theorem.

Hits (in millions) returned by google:

X	that	whether	what	how	who	why
"know X"	574	28	592	490	112	113
"knows X"	50.7	0.51	61.4	86.3	8.48	3.55

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Linguistically: factivity, exhaustivity, concealed questions

Philosophically: reducible to "knowledge-that"?

Logically: how to reason about "knowing-wh"?

Computationally: efficient representation and reasoning

WE INDEED WANT TO KNOW WHY /HOW /WHAT....

Shubhendu Sharma
How to grow a forest in your backyard
Posted Jul 2016
Rated Inspiring, Informative

Emma Marris
Nature is everywhere — we just need to learn to see it
Posted Jul 2016
Rated Inspiring, Informative

Eric Haseltine
What will be the next big scientific breakthrough?
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3 lessons on success from an Arab businesswoman
Posted Jul 2016
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Elise Roy
When we design for disability, we all benefit
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Safwat Saleem
Why I keep speaking up, even when people mock my accent
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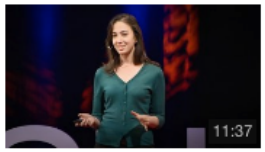

Julia Galef
Why you think you're right — even if you're wrong
Posted Jun 2016
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Blaise Agüera y Arcas
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How to fix a broken education system ... without any more money
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Why genetic research must be more diverse
Posted Jun 2016
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I survived a terrorist attack. Here's what I learned
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It helps to go back to the starting point of epistemic logic.

BEYOND "KNOWING THAT": HINTIKKA'S EARLY WORK

"knowing who" was discussed by Hintikka (1962) in terms of first-order modal logic: $\exists x\mathcal{K}(Mary \approx x)$, i.e., knowing the answer of the embedded question.

Hintikka used epistemic logic to understand questions. E.g, consider the question Q : "Who murdered Mary?"

- The *presupposition* of Q is $\mathcal{K}\exists xM(x, Mary)$.
- The *desideratum* of Q is $\exists x\mathcal{K}M(x, Mary)$.
- One possible answer to Q is $M(John, Mary)$.
- *Conclusiveness* of the answer requires $\exists x\mathcal{K}(John \approx x)$.
- Conclusive answers realize the desideratum ($\mathcal{K}\exists x$ to $\exists x\mathcal{K}$).

KNOW-WH INSPIRED HINTIKKA TO GO FURTHER

- This interest in the relationship between questions and knowledge also led Hintikka to the pursuit of a *Socratic epistemology* that weighs *knowledge acquisition* more importantly than *knowledge justification*.
- The attempt to avoid quantifying higher-order entities in formalizing sentences like "I know whom every young mother should trust" leads Hintikka to the core idea of *independent friendly logic* (from $\exists f \mathcal{K} \forall x (M(x) \rightarrow T(x, f(x)))$ to $\mathcal{K} \forall x (\exists y / \mathcal{K})(M(x) \rightarrow T(x, y))$).

See my survey paper for the early contributions of Hintikka to logics of know-wh.

In *Meaning and Necessity* (1947), Carnap remarked:

> *Any system of modal logic without quantification is of interest only as a basis for a wider system including quantification. If such a wider system were found to be impossible, logicians would probably abandon modal logic entirely.*

However, it seems that history went exactly the other way around.

MANY THINGS CAN BE DONE IN FIRST-ORDER MODAL LOGIC

First-order modal logic is infamous for:

- issues in the semantics
- *quantifying-in* and substitution
- ambiguity: *de re* vs. *de dicto*
- incompleteness
- lack of Craig's interpolation
- undecidablility (hard to find useful decidable fragments)
- ....

At the same time, propositional modal logic is too successful...

In the latest *Handbook of Epistemic Logic*, there is hardly anything about first-order epistemic logic.

RECENT DEVELOPMENTS FOR FO EPISTEMIC LOGIC

A slightly out-dated survey in Gochet and Gribomont (2006)

Mostly application-driven (not an exhaustive list):

- about games: Kaneko and Nagashima (1996)
- about cryptographic knowledge: Cohen and Dam (2007)
- about security protocols: Belardinelli and Lomuscio (2011)
- (un)decidability: Wolter (2000), Sturm et al (2000)
- *de dicto* vs. *de re*: distinction Corsi and Orlandelli (2011)
- second-order epistemic logic: Belardinelli and van der Hoek (2015, 2016)
- ...

OUR "MINIMALIST" APPROACH

Instead of trying to tame the infamous full quantified epistemic logic, we ...

- take a know-wh construction as a single modality, e.g., pack $\exists x K_i(Mary \approx x)$ into $Kwho_i Mary$;
- give some intuitive semantics for certain subtypes/interpretations of knowing-wh;
- axiomatize logics with (combinations of) new operators;
- dynamify those logics with new updates of knowledge;
- add new group notions, and dynamic or temporal aspects.
- automate the inferences;
- (probably) come back to philosophy and linguistics with new insights and questions.

THE (POTENTIAL) ADVANTAGES OF MODAL LOGICS OF "KNOWING-WH"

- Natural and succinct to express the desired properties;
- Limited expressive power and moderate complexity (secret of success of modal logic);
- Capture the essence of the relevant reasoning by axioms;
- Stay technically neutral in some philosophical debates;
- Formal notion of consistency of knowledge bases;

BEYOND KNOWING THAT: (TECHNICAL) DIFFICULTIES

- not *normal*:
  - $\not\vdash \mathcal{K}w(p \to q) \land \mathcal{K}w\, p \to Kw\, q$
  - $\not\vdash \mathcal{K}h\varphi \land \mathcal{K}h\psi \to \mathcal{K}h(\varphi \land \psi)$
  - $\vdash \varphi \not\Rightarrow \vdash \mathcal{K}y\varphi$
- not strictly weaker: $\vdash Kw\varphi \leftrightarrow Kw\neg\varphi$;
- combinations of quantifiers and modalities: $\exists x\mathcal{K}\varphi(x)$;
- the axioms depend on the special schema of φ essentially;
- weak language vs. rich model: hard to axiomatize;
- fragments of FO/SO-modal language;

SOME PEOPLE INVOLVED SO FAR

- Jie Fan, Yanjun Li, Tszyuen Lau, Shihao Xiong, Yifeng Ding, Tao Gu, Chao Xu, Xingchi Su, Jixin Liu, Zhouhang Zhou ...;
- Hans van Ditmarsch, Malvin Gattinger, Jan van Eijck, Alexandru Baltag, Andreas Herzig, Raul Fervari, Thomas Studer, Pavel Naumov, Jia Tao, Valentin Goranko, Fernando Velázquez-Quesada, Jeremy Seligman...

SOME RESULTS

- Knowing whether: [Fan, W.& van Ditmarsch: AiML14, RSL15] [Fan & vD: ICLA15, JANCL16], [Fan 17]
- Knowing what: [W. & Fan: IJCAI13, AiML14][Gu & W. AiML16], [Baltag, AiML16] [van Eijck, Gattinger, W. ICLA17]
- Knowing how: [W. LORI15], [W. Synthese17], [Li, W. ICLA17][Herzig, Fervari, Li, W. IJCAI17], [Fervari, Velázquez-Quesada, W. SR17][Naumov & Tao TARK17...]
- Knowing why: [Xu, W., Studer 16]
- Knowing who: [W., Seligman: AiML18]
- Special column in *Studies in Logic* by Fan, Li, Ding.

CHARACTERISTIC FEATURE

How to distinguish the work in this line and other related work in the literature?

Whether it uses a single modality for know-wh, instead of breaking it down into quantifiers, normal modalities, questions, predicates and so on

EXAMPLE: KNOWING HOW [FERVARI, HERZIG, LI, W. IJCAI17]

TAUT	all axioms of propositional logic	MP	$\dfrac{\varphi, \varphi \to \psi}{\psi}$
DISTK	$\mathcal{K}p \wedge \mathcal{K}(p \to q) \to \mathcal{K}q$	NECK	$\dfrac{\varphi}{\mathcal{K}\varphi}$
T	$\mathcal{K}p \to p$	EQREPKh	$\dfrac{\varphi \to \psi}{\mathcal{K}h\varphi \to \mathcal{K}h\psi}$
4	$\mathcal{K}p \to \mathcal{K}\mathcal{K}p$	SUB	$\dfrac{\varphi(p)}{\varphi[\psi/p]}$
5	$\neg\mathcal{K}p \to \mathcal{K}\neg\mathcal{K}p$		
AxKtoKh	$\mathcal{K}p \to \mathcal{K}hp$		
AxKhtoKKh	$\mathcal{K}hp \to \mathcal{K}\mathcal{K}hp$		
AxKhtoKhK	$\mathcal{K}\mathcal{K}hp \to \mathcal{K}h\mathcal{K}p$		
AxKhKh	$\mathcal{K}h\mathcal{K}hp \to \mathcal{K}hp$		
AxKhbot	$\neg\mathcal{K}h\bot$		

CONNECTIONS TO EXISTING LOGICS AND LINGUISTIC THEORIES

Classification by question words:

- Knowing whether: non-contingency logic, ignorance logic
- Knowing what: weakly aggregative logic, dependence logic
- Knowing how: game Logic, alternating temporal logic
- Knowing why: quantified justification Logic

Classification by logical forms:

- *Mention-some*: e.g., *knowing how/why*... $\exists x \mathcal{K} \varphi(x)$
- *Mention-all* (strongly exhaustive reading): e.g., I *know who* came to the party... $\forall x(\mathcal{K}\varphi(x) \vee \mathcal{K}\neg\varphi(x))$
- In-between: *know-value* $\exists x(\mathcal{K}\, c \approx x) \leftrightarrow \forall x(\mathcal{K}\, c \approx x \vee \mathcal{K}\, c \not\approx x)$

EPISTEMIC LOGIC: FORM ONE TO MANY

(Routine) research questions:

- Model theory, proof theory, computational complexity
- Group knowledge
- Logical omniscience
- Natural dynamics
- Applications

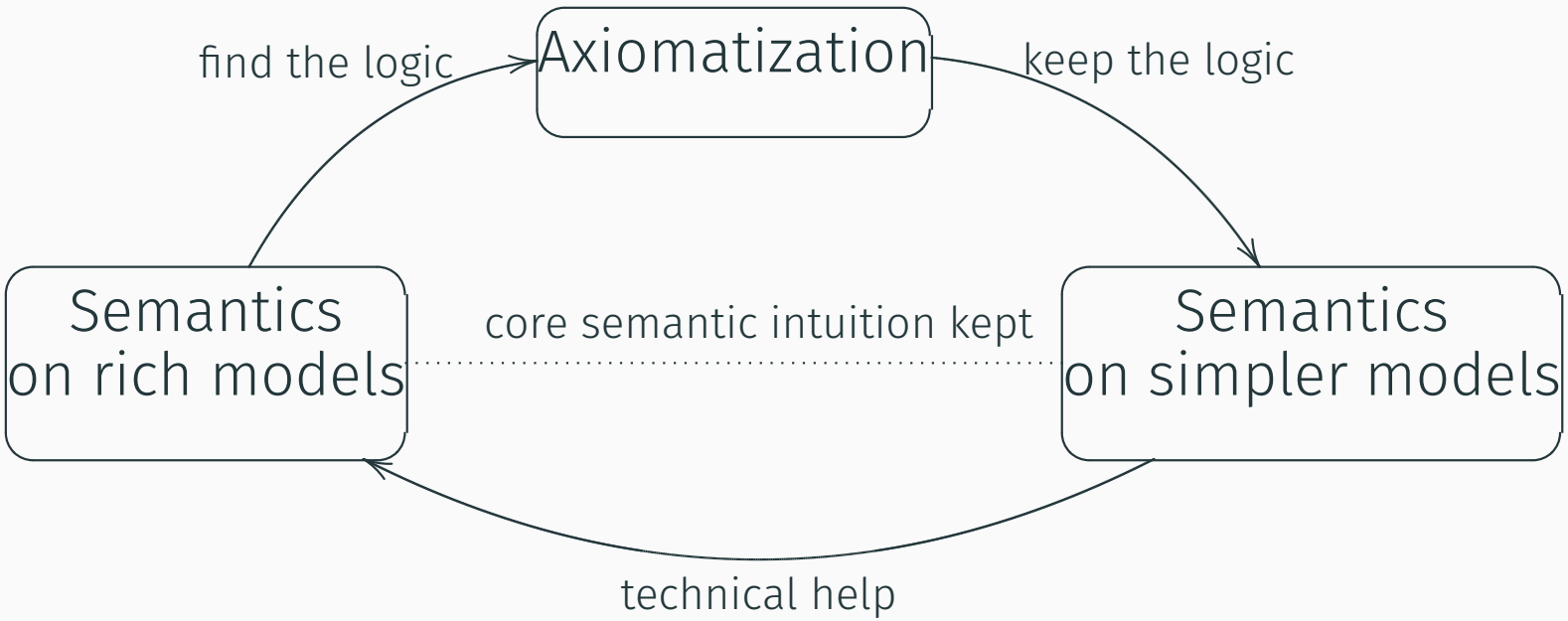
New questions:

- Interactions of different knowledge expressions
- Simplification of semantics

SIMPLIFY THE SEMANTICS WHILE KEEPING THE LOGIC

Common difficulties: weak language vs. rich semantics

To restore the balance between the language and model:



DISADVANTAGES OF THOSE CONCRETE LOGICS

Disadvantages from a linguistic point of view:

- Compositionality
- Uniformity
- Expressivity

Disadvantages in terms of knowledge representation:

- Propositional epistemic logic is not really about the *content* of knowledge!

A question: how to explain the decidability of those logics?

TOWARDS A GENERAL NEW FRAMEWORK

What we are after:

- Expressive enough: covering the essence of those non-standard epistemic logics
- Not too much: sharing most good properties of propositional modal logic

Uniformity, compositionality, expressvity, computability: we want a predicate modal framework like the propositional modal logic

A NEW FRAMEWORK FOR PREDICATE EPISTEMIC LOGIC

Inspired by the concrete know-wh logics, we introduce the bundle modalities into the predicate modal language:

- pack $\exists x\mathcal{K}$ into a *bundle* modality (mention-some)
- pack $\forall x\mathcal{K}w$ into a *bundle* modality (mention-all)

You can also come up with your favourite bundles.

We obtain some nice and powerful fragments of first-order modal logic.

A NEW FRAMEWORK FOR PREDICATE EPISTEMIC LOGIC

Example: epistemic language of mention-some [W. TARK17]:

$$\varphi ::= P\overline{x} \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \exists x\mathcal{K}\varphi$$

$\exists x\mathcal{K}\varphi$: I know some thing such that φ

- "I know a theorem of which I do not know any proof": $\exists x\mathcal{K}\neg\exists y\mathcal{K}Prove(y,x)$
- "i knows a country which j knows its capital": $\exists x\mathcal{K}_i\exists y\mathcal{K}_j Capital(y,x)$

PRICE TO PAY?

Some bundles are very good in terms of model theoretical and computational properties.

E.g., for the $\exists x\mathcal{K}$ bundle on top of a predicate logical language

- PSPACE-complete (as propositional modal logic) over constant/increasing-domian models (no need to restrict the arity of predicates or number of variables)
- van Benthem-Rosen characterization theorems w.r.t. full first-order modal logic

A gift given by natural language to logic.

About the course

GOALS OF THIS COURSE

- attention to logical aspects of know-wh expressions
- awareness of some conceptual/technical questions
- knowledge of relevant techniques for bundle modalities
- familiarity with fragments of first-order modal logic
- thinking process behind example logics
- for me, to also learn from you

DISCLAIMER

This is a logic course. This course is *not* about:

- formal semantics for natural language
- epistemology

Some other warnings:

- idealized agents cf. Rineke's course
- not that much about multi-agent cf. Valentin's course
- not that much about dynamics cf. others' courses
- not really about belief cf. Alexandru & Sonja's course
- no probability cf. Thomas & Krzysztof's course
- mainly Kripke semantics cf. Adam, Alexandru & Sonja's

SCHEDULE OF THE COURSE

- Today: The general picture
- Day 2: Knowing whether
- Day 3: Knowing what
- Day 4: Knowing how
- Day 5: The general framework and future directions
- Other topics: knowing why, knowing who ...

SOME PRACTICAL ISSUES

Slides and pointers will be updated at: `http://www.phil.pku.edu.cn/personal/wangyj/NASSLLI18`

- Model theory of propositional modal logic: e.g., first four chapters of *Modal Logic* by Blackburn, de Rijke, & Venema;
- Some knowledge of first-order logic;
- Mainly focus on axiomatizations;
- Ideas rather than technical details;
- Some familiarity with epistemic logic is definitely helpful, but it is also OK otherwise (other epistemic logic courses)

Questions?

Whether? What? How? Why?

http://www.phil.pku.edu.cn/personal/wangyj/NASSLLI18

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