BEYOND "KNOWING THAT" (V)

A GENERAL FRAMEWORK OF PREDICATE MODAL LOGIC

Yanjing Wang
Department of Philosophy, Peking University
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www.wangyanjing.com

Summary for now

IN THE PAST FEW DAYS

We have seen:

- Logics of knowing whether (non-contingency)
- Logics of knowing what (value)
- Logics of (goal-directed) knowing how

We *never* claimed THE logic of know-wh.

WHAT WE HAVEN'T COVERED

- A logic of knowing why [Xu, Wang, Studer]
  - based on quantified justification logic $\mathcal{K}y\varphi := \exists t\mathcal{K}(t:\varphi)$
  - no necessitation rule
  - troubles: introspections for $\mathcal{K}y$, K axiom
  - another Chinese teaching: $\mathcal{K}\varphi \to \mathcal{K}y\varphi$.
- Towards a logic of knowing who [Wang & Seligman AiML18]
  - when if names are not common knowledge ...
  - based on quantifier-free term modal predicate logic
  - a knows someone named b knows it needs help but a does not know who b is: $[x := a]\mathcal{K}_a\mathcal{K}_b H(x) \wedge \neg[x := b]\mathcal{K}_a(x \approx b)$
  - See [Aloni 16] for handling various interpretations.

REVIEW: THE ADVANTAGES OF MODAL LOGICS OF “KNOWING-WH”

- Natural and succinct to express the desired properties;
- Limited expressive power and moderate complexity (secret of success of modal logic);
- Capture the essence of the relevant reasoning by axioms;
- Formal notion of consistency of knowledge bases.

CONNECTIONS TO EXISTING LOGICS

Classification by question words:

- Knowing whether: non-contingency logic, ignorance logic
- Knowing what: weakly aggregative logic, dependence logic
- Knowing how: game Logic, alternating temporal logic
- Knowing why: quantified justification Logic
- Knowing who: term modal logic, dynamic logic

Knowing-wh can be used as a central thread to connect a large chunk of philosophical logics.

They also connect well with neighbourhood semantics but you can do non-normal modal logic with intuitive Kripke models!

EPISTEMIC LOGIC: FORM ONE TO MANY

(Routine) research questions:

- Model theory, proof theory, computational complexity
- Group knowledge
- Logical omniscience
- Natural dynamics
- Applications

New questions:

- Interactions of different knowledge expressions
- Simplification of semantics

BEYOND BEYOND KNOWING THAT

Disadvantages from a linguistic point of view:

- Compositionality
- Uniformity
- Expressivity

Disadvantages in terms of knowledge representation:

- Propositional epistemic logic is not really about the *content* of knowledge!

Logic of mention-some/-all

AIMING AT A GENERAL NEW FOUNDATION

What we are after:

- Expressive enough: covering the essence of those non-standard epistemic logics
- Not too much: sharing most good properties of propositional modal logic
- Avoid philosophical controversies, if possible

RECALL THE UNDERLYING LOGICAL PATTERNS

Classification by logical forms:

- *Mention-some*: e.g., *knowing how/why...* $\exists x \mathcal{K} \varphi(x)$
- *Mention-all* (strongly exhaustive reading): e.g., I *know who* came to the party... $\forall x \mathcal{K}w \varphi(x)$
- In-between: *know-value* $\exists x \mathcal{K}(c \approx x) \leftrightarrow \forall x \mathcal{K}w(c \approx x)$

A NEW GENERAL FRAMEWORK [WANG TARK17]

Definition (Language $\mathsf{MLMS}^{\approx}$)

Given set of variables X and set of predicate symbols Ps,

$$\varphi ::= x \approx y \mid P\overline{x} \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \Box^x \varphi$$

where $x, y \in \mathrm{X}$, $P \in \mathrm{Ps}$. Call the *equality-free* fragment MLMS.

$\Box^x$ is actually $\exists x \Box$. It is dual $\Diamond^x$ is essentially $\forall x \Diamond$.

$\Box^x \varphi$ reads "I know some x such that $\varphi(x)$".

Modal logic of *mention some* (vs. *mention all*)

EXPRESSIVITY

- Knowing-wh (*mention-some*): $\Box^x \varphi(x)$
- "I know a theorem of which I do not know any proof": $\Box^x \neg \Box^y Prove(y, x)$
- "*i* knows a country which *j* knows its capital": $\Box_i^x \Box_j^y Capital(y, x)$

FIRST-ORDER KRIPKE SEMANTICS

Definition (First-order Kripke Model)

An increasing domain model $\mathcal{M} = \langle W, D, \delta, R, \rho \rangle$ where:

- W is a non-empty set.
- D is a non-empty set (local domains are its subsets)
- $R \in 2^{W \times W}$ is a binary relation over W.
- $\delta : W \to 2^D$ assigns to each $w \in W$ a *non-empty* local domain s.t. wRv implies $\delta(w) \subseteq \delta(v)$ for any $w, v \in W$.
- $\rho : \text{Ps} \times W \to \bigcup_{n \in \omega} 2^{D^n}$ such that ρ assigns each n-ary predicate on each world an n-ary relation on D.

We write $D_w^{\mathcal{M}}$ for the local domain $\delta(w)$ in $\mathcal{M}$.

SEMANTICS

Definition ($\exists\Box$ Semantics)

$$\begin{aligned}\mathcal{M}, w, \sigma \vDash \Box^x \varphi \quad &\Leftrightarrow \quad \text{there exists an } a \in D_w^{\mathcal{M}} \text{ such that} \\ &\qquad \mathcal{M}, v, \sigma[x \mapsto a] \vDash \varphi \text{ for all } v \text{ s.t. } wRv \\ &\Leftrightarrow \quad \text{there exists an } a \in D_w^{\mathcal{M}} \text{ such that} \\ &\qquad \mathcal{M}, w, \sigma[x \mapsto a] \vDash \Box\varphi\end{aligned}$$

MLMS$^{\approx}$ is indeed an extension of propositional modal logic:

$\vDash \Box\varphi \leftrightarrow \Box^x\varphi$ (given x is not free in φ).

An MLMS$^{\approx}$ formula φ is *satisfiable* if there is an increasing domain pointed model $\mathcal{M}, w$ and an assignment σ such that $\mathcal{M}, w, \sigma \vDash \varphi$ and $\sigma(x) \in D_w^{\mathcal{M}}$ for all $x \in X$.

∃□-bisimulation (inspired by monotonic and obj-world bis)

Given $\mathcal{M}$ and $\mathcal{N}$, non-empty $Z \subseteq (W_\mathcal{M} \times D^*_\mathcal{M}) \times (W_\mathcal{N} \times D^*_\mathcal{N})$ is call an *∃□-bisimulation*, if for every $((w, \overline{a}), (v, \overline{b})) \in Z$ such that $|\overline{a}| = |\overline{b}|$ the following holds (we write $w\overline{a}$ for $(w, \overline{a})$):

PISO $\overline{a}$ and $\overline{b}$ form a partial isomorphism w.r.t. identity and interpretations of predicates at w and v respectively.

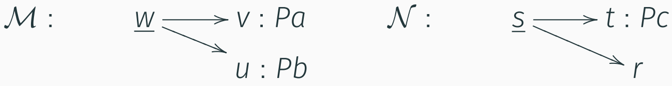
∃□Zig For any $c \in D^\mathcal{M}_w$, there is a $d \in D^\mathcal{N}_v$ such that for any $v' \in W_\mathcal{N}$ if vRv' then there exists w' in $W_\mathcal{M}$ such that wRw' and $w'\overline{a}cZv'\overline{b}d$. $(\forall^{object}_\mathcal{M} \exists^{object}_\mathcal{N} \forall^{world}_\mathcal{N} \exists^{world}_\mathcal{M})$

∃□Zag Symmetric to ∃□Zig.

We say $\mathcal{M}, w\overline{a}$ and $\mathcal{N}, v\overline{b}$ are *∃□-bisimilar* ($\mathcal{M}, w\overline{a} \leftrightarroweq_{\exists\Box} \mathcal{N}, v\overline{b}$) if $|a| = |b|$ and there is an ∃□-bisimulation linking $w\overline{a}$ and $v\overline{b}$.

EXAMPLE

Consider the *constant domain* models $\mathcal{M}$ and $\mathcal{N}$:



where $D^{\mathcal{M}} = \{a, b\}$, $D^{\mathcal{N}} = \{c\}$. Suppose P is the only predicate, we can show that $\mathcal{M}, w \underline{\leftrightarrow}_{\exists\Box} \mathcal{N}, s$ by an $\exists\Box$-bisimulation Z:

$$\{(w, s), (va, tc), (ub, tc), (vb, rc), (ua, rc)\}$$

Note that $\exists\Box$Zig and $\exists\Box$Zag hold trivially for $w\bar{a}$ and $v\bar{b}$ if w and v *do not* have any successor, based on the fact that local domains are non-empty by definition.

LIMITED EXPRESSIVE POWER

Theorem

$\mathcal{M}, w\bar{a} \leftrightarrows_{\exists\Box} \mathcal{N}, v\bar{b}$ *then* $\mathcal{M}, w\bar{a} \equiv_{\mathrm{MLMS}^{\approx}} \mathcal{N}, v\bar{b}$.

Proposition

$\Box\exists x Px$, $\exists x \Diamond Px$ *and* $\Diamond\exists x Px$ *are not expressible in* $\mathrm{MLMS}^{\approx}$.

Theorem

For $\exists\Box$*-saturated models* $\mathcal{M}, \mathcal{N}$ *and* $|\bar{a}| = |\bar{b}|$:

$$\mathcal{M}, w\bar{a} \leftrightarrows_{\exists\Box} \mathcal{N}, v\bar{b} \Leftrightarrow \mathcal{M}, w\bar{a} \equiv_{\mathrm{MLMS}^{\approx}} \mathcal{N}, v\bar{b}$$

SOME NICE PROPERTIES

Results

- van-Benthem-Rosen Characterization w.r.t. FO-modal logic
- A satisfiable MLMS formula has a *finite tree* model.
- A tableau method for satisfiability of MLMS
- Satisfiability checking of MLMS is PSPACE-complete
- MLMS does not distinguish increasing /constant domain

MLMS behaves like the basic propositional modal logic but much more powerful.

TABLEAUX (CAN BE VIEWED AS A SATISFIABILITY GAME)

We start from negated normal form:

$$\varphi ::= P\overline{x} \mid \neg P\overline{x} \mid (\varphi \wedge \varphi) \mid (\varphi \vee \varphi) \mid \Box^x \varphi \mid \Diamond^x \varphi$$

$$\frac{w : \varphi_1 \vee \varphi_2, \Gamma, \sigma}{w : \varphi_1, \Gamma, \sigma \mid w : \varphi_2, \Gamma, \sigma}\ (\vee) \qquad \frac{w : \varphi_1 \wedge \varphi_2, \Gamma, \sigma}{w : \varphi_1, \varphi_2, \Gamma, \sigma}\ (\wedge)$$

Given $n \geq 0, m \geq 1$:

$$\frac{w : \Box^{x_1}\varphi_1, \ldots, \Box^{x_n}\varphi_n, \Diamond^{y_1}\psi_1, \ldots, \Diamond^{y_m}\psi_m, l_1 \ldots l_k, \sigma}{\{(wv^{y}_{y_i} : \{\varphi_j \mid 1 \leq j \leq n\}, \psi_i[y/y_i], \sigma') \mid y \in Dom(\sigma'), i \in [1, m]\}}\ (\mathrm{BR})$$

Given $n \geq 1, k \geq 0$:

$$\frac{w : \Box^{x_1}\varphi_1, \ldots, \Box^{x_n}\varphi_n, l_1 \ldots l_k, \sigma}{w : l_1 \ldots l_k, \sigma}\ (\mathrm{END})$$

where $\sigma' = \sigma \cup \{(x_j, x_j) \mid j \in [1, n]\}$ and $l_k \in lit$ (the literals).

AN EXAMPLE

Is $\Box^x(Px \vee Qx) \wedge \Diamond^y \neg Qy \wedge \neg Pz$ satisfiable?

$$w : \{\Box^x(Px \vee Qx) \wedge \Diamond^y \neg Qy \wedge \neg Pz, \{(z,z)\} \quad (\wedge) \times 2$$

$$w : \{\Box^x(Px \vee Qx), \Diamond^y \neg Qy, \neg Pz\}\{(z,z)\} \quad \text{(BR)}$$

$$wv^x_y : \{Px \vee Qx, \neg Qx\}, \{(x,x),(z,z)\} \quad (\vee) \quad wv^z_y : \{Px \vee Qx, \neg Qz\}, \{(x,x),(z,z)\}$$

$$wv^x_y : \{Px, \neg Qx\}, \{(x,x),(z,z)\} \qquad wv^z_y : \{Qx, \neg Qz\}, \{(x,x),(z,z)\}$$

A COMPLETE EPISTEMIC LOGIC OVER S5 MODELS

Over S5 (constant-domain) models, MLMS is very powerful, it can also express *mention-all* by $\Diamond^x(\Box\varphi \vee \Box\neg\varphi)$. Moreover, $\forall x\Box$, $\Box\forall x$, $\exists\overline{x}\Box$ are expressible but $\Box\exists x$ is not.

Axioms	
TAUT	all axioms of propositional logic
DISTK	$\Box(\varphi \to \psi) \to \Box\varphi \to \Box\psi$
T	$\Box\varphi \to \varphi$
4MS	$\Box^x\varphi \to \Box\Box^x\varphi$
5MS	$\neg\Box^x\varphi \to \Box\neg\Box^x\varphi$
KtoMS	$\Box(\varphi[y/x]) \to \Box^x\varphi$ (if $\varphi[y/x]$ is admissible)
MStoK	$\Box^x\varphi \to \Box\varphi$ (if $x \notin FV(\varphi)$)
MStoMSK	$\Box^x\varphi \to \Box^x\Box\varphi$
KT	$\Box\top$

Rules:

MP	$\dfrac{\varphi, \varphi \to \psi}{\psi}$
MONOMS	$\dfrac{\vdash \varphi \to \psi}{\vdash \Box^x\varphi \to \Box^x\psi}$

For MLMS$^{\approx}$ we also need **ID** : $x \approx x$ and **SUBID** : $x \approx y \to (\varphi \to \psi)$.

We can derive **KEQ** : $x \approx y \to \Box(x \approx y)$ and **KNEQ** : $x \not\approx y \to \Box(x \not\approx y)$.

COMPARISON: KNOWING HOW [FERVARI, HERZIG, LI, W. IJCAI17]

TAUT	all axioms of propositional logic	MP	$\dfrac{\varphi, \varphi \to \psi}{\psi}$
DISTK	$\mathcal{K}p \land \mathcal{K}(p \to q) \to \mathcal{K}q$	NECK	$\dfrac{\varphi}{\mathcal{K}\varphi}$
T	$\mathcal{K}p \to p$	EQREPKh	$\dfrac{\varphi \to \psi}{\mathcal{K}h\varphi \to \mathcal{K}h\psi}$
4	$\mathcal{K}p \to \mathcal{K}\mathcal{K}p$	SUB	$\dfrac{\varphi(p)}{\varphi[\psi/p]}$
5	$\neg\mathcal{K}p \to \mathcal{K}\neg\mathcal{K}p$		
AxKtoKh	$\mathcal{K}p \to \mathcal{K}hp$		
AxKhtoKhK	$\mathcal{K}hp \to \mathcal{K}h\mathcal{K}p$		
AxKhtoKKh	$\mathcal{K}hp \to \mathcal{K}\mathcal{K}hp$		
AxKhKh	$\mathcal{K}h\mathcal{K}hp \to \mathcal{K}hp$		
AxKhbot	$\neg\mathcal{K}h\bot$		

understanding the (un)decidability

First-order modal logic is infamous for its "robust" undecidability. Known (un)decidable fragments of FOML:

Language	Model	Decidability	Ref
x, y, p, P^1	inc-D, cons-D	undecidable	[KKZ05, GS93]
$x, y, \Box_i$, single P^1	inc-D, cons-D	undecidable	[RS17]
single x	inc-D, cons-D	decidable	[Seg73, FS+78]
$x, y / P^1 /$ GF + $\Box_i(x)$	inc-D, cons-D	decidable	[WZ01]

The bundle modality idea may give us a new way to discover nice fragments.

UNDERSTANDING THE (UN)DECIDABILITY

- Over constant/increasing domain models, MLMS is PSPACE-complete.
- Over S5 models, MLMS is *not decidable* in general. We can code FOL with three variables and binary relations.
- Most existing propositional knowing-wh logics are *one-variable* fragments of FO-modal logic.

WHAT ABOUT OTHER BUNDLES?

Recent results [Padmanabha, Ramanujam, Wang]

Language	Domain	Decidability	Remark
$\forall\Box$, P^1	cons-D	undecidable	
$\Box\forall$, P^1	cons-D	undecidable	
$\exists\Box$, P	cons-D	decidable	PSPACE-complete
$\exists\Box, \forall\Box, P$	inc-D	decidable	PSPACE-complete

ONGOING WORK

- Logic of mention-all [Zhou BA thesis18]
- Adding equality, constants and function symbols...

Conclusions

CONCLUSIONS

- Interesting knowledge statements beyond know-that
- Various interpretations for them
- New modal operators by packing things together
- Non-normal modal logic based on Kripke models
- Techniques for imbalance between model and language
- Techniques for simplifying semantics
- Decidable FOML fragments using bundles to restrict the behaviour of quantifiers.

Sometimes natural language also tells us sth about complexity.

FUTURE WORK

- Theoretical properties of the existing know-wh logics.
- Other semantics for different interpretations of know-wh.
- Study multi-agent and group notions of know-wh.
- Combination of different know-wh operators.
- Systematic study of various bundle fragments.
- Applications in AI and philosophy.

FROM HINTIKKA (1983)

> *I once read about a cannibal tribe in which nobody could become a chieftain without disposing of one of the earlier ones and eating him. It seems to me sometimes that philosophers must be descendants of that tribe. When a philosopher develops a new theory, it almost invariably seems more important to him to use it to try to clobber an earlier one rather than to try to see if the two are perhaps complementary - and to see what there is, perhaps, to be learned from the earlier theory.*

Let's have some more space thus everyone can play without eating each other.

BEYOND KNOWING THAT: JOIN US BEFORE IT IS TOO CROWDED!

There are lots of new things going on in China. Come and see.

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